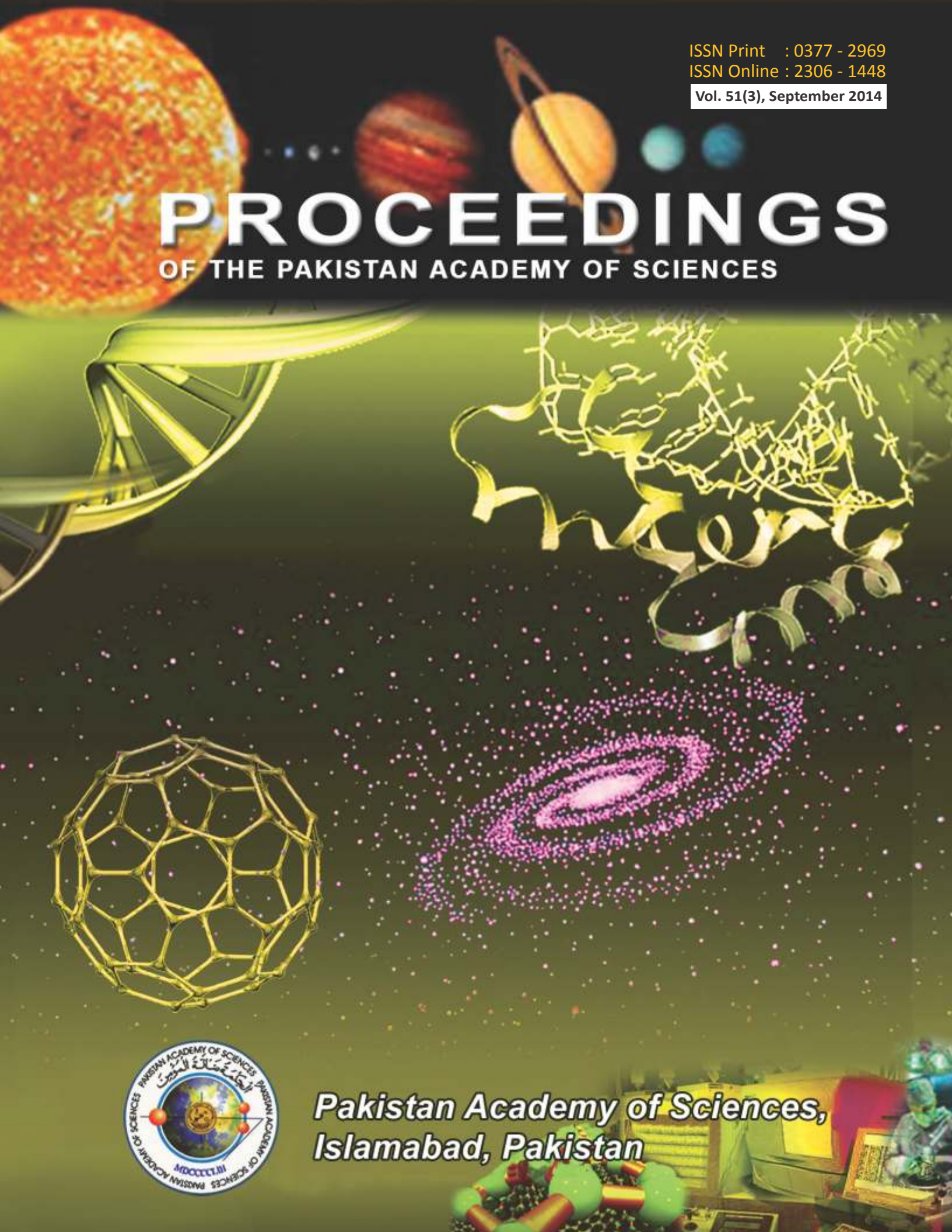
ISSN Print : 0377 - 2969
ISSN Online : 2306 - 1448
Vol. 51(3), September 2014

PROCEEDINGS
OF THE PAKISTAN ACADEMY OF SCIENCES



Pakistan Academy of Sciences,
Islamabad, Pakistan

Proceedings of the Pakistan Academy of Sciences, published since 1964, is official journal of the Academy, published quarterly. This open access journal publishes research papers and reviews in basic and applied sciences. All papers are peer reviewed. Authors are not required to be Fellows or Members of the Academy, or citizens of Pakistan.

Annual Subscription: **Pakistan:** Institutions, Rupees 2000/- ; Individuals, Rupees 1000/-
Other Countries: US$ 100.00 (includes air-lifted overseas delivery)

HEC Recognized, Category X; PM&DC Recognized

Published by **Pakistan Academy of Sciences**, 3 Constitution Avenue, G-5/2, Islamabad, Pakistan
Tel: 92-5 1-920 7140 & 921 5478; **Fax:** 92-51-920 6770; **Website: www.paspk.org; Email:** pas.editor@gmail.com

Printed at **PanGraphics (Pvt) Ltd.,** No. 1, I & T Centre, G-7/l, Islamabad, Pakistan
Tel: 92-51-220 2272, 220 2449; **Fax**: 92-51-220 2450; **Email**: pangraph@isb.comsats.net.pk

Proceedings
OF THE PAKISTAN ACADEMY OF SCIENCES

CONTENTS

Volume 51, No. 3, September 2014 **Page**

Submission of Manuscripts: Manuscripts may be submitted online at www.paspk.org or as E-mail attachment. Authors must consult the ***Instructions for Authors*** at the end of this issue and at the Website: www.paspk.org

Proceedings of the Pakistan Academy of Sciences 51 (3): 187–192 (2014)

ISSN: 0377 - 2969 (print), 2306 - 1448 (online)

Pakistan Academy of Sciences

Research Article

A Characteristic Study of Exponential Distribution Technique in a Flowshop using Taillard Benchmark Problems

R. Pugazhenthi* and M. Anthony Xavior

School of Mechanical and Building Sciences, VIT University,
Vellore, Tamil Nadu 632014, India

Abstract: The main objective of many authors is to find an optimum sequence, which can provide minimum makespan in a Permutation Flow Shop (PFS). In this venture, a new Exponential Distribution Technique (EPDT) is proposed, under the mathematical and computational features. This paper deals with characteristic study of EPDT over the existing algorithms. The taillard benchmark problems are solved for the general comparison. This analysis have been tabulated and graphically represented along with the cumulative performance of it. The solution of this work have shown that Exponential Distribution Technique has better performance in finding an optimal sequence in a permutation flow shop.

Keywords: Flowshop, makespan, exponential distribution technique (EPDT), optimal sequence

1. INTRODUCTION

In production management, a scheduling problem is defined as work time hypothesis regarding assignment of resources like raw materials, machines, etc. In a complex and dynamic production environment, scheduling is an extremely important issue. Scheduling deals with the allocation of available resources to tasks over time.

Permutation flow shop problem is a special case of the flow shop problem. A possible constraint in the flow shop environment is that the queues for each machine operate according to the FIFO discipline. This implies that the order in which the jobs go through the first machine is the same throughout the system. The most important property of PFS is deciding the job sequence on the first machine because once it is decided, all the jobs follow the same sequence on each machine throughout the sys with n-jobs, there are n! solutions that are independent of machine numbers.

The various algorithmic methods have been suggested to obtain optimum makespan. While the makespan is abridged, the corresponding parameters such as idle time, waiting time, total flow time etc., get distorted. During the last six decades, the flowshop sequencing problem has been the active centre of attention for many researchers.

In 1954, a simple algorithm was given by Johnson [1], for flowshop scheduling problems in the order of 'n' jobs in '2' machines. This work was further developed by Ignall and Scharge [2].

In the 1965's Palmer [3, 4] have been the first to propose heuristic procedures. The first significant work in the development of an efficient heuristic is due to Campell, Dudek and Smith. Their algorithm consists essentially in splitting the given m-machine problem into a series of an equivalent two-machine flow shop problem and solving by Johnson's rule.

A simple modification and extension of Palmer's heuristic was carried out by Hundal and Rajagopal [5], two sets of indices have been proposed and three sequences were obtained. In 1977, Dannenbring [6] has developed a procedure called 'rapid access', which attempts to combine

Received, November 2013; Accepted, August 2014
* Corresponding author: R. Pugazhenthi; Email: Puga14@gmail.com

the advantages of Palmer's slope index and CDS procedures. Though the procedure by Dannenbring has found to yield a better quality solution than those by Palmer's and CDS methods; it requires much more computational effort.

During 1980s, King and Speeches [7] treated the makespan problem as equivalent to that of minimizing total delay and run-out delay. They have proposed heuristics that aim at matching the two consecutive job time-block profiles by considering these delays. One of the heuristics turns out to be better than the CDS heuristic.

In 1982, Stinson et al [8] has proposed a radically different approach. They treated the makespan oriented problem as a travelling salesman problem and developed a procedure in two steps. The heuristic solution is found to yield a better quality solution than those by Palmer and CDS methods at the cost of increased computational effort.

2. STUDY METHODOLOGIES

2.1 Gupta Method (GUPTA)

Gupta [9] suggested another heuristic which is similar to Palmer's heuristic using a slope index in a different manner. In this method, the effect of first and last machine is high in predicting slope index. This effect causes the demerits in finding the best sequence.

Step 1: Calculate the value of the function associated with job i, f (i), as follows

$$f(i) = \frac{A}{Min(t_{ij} + t_{i,j+1})} \text{ for j =1, 2, …, (m-1).} \quad (1)$$

Where, A = 1 if $t_{iM} \leq t_{i1}$,

A = -1 otherwise.

Step 2: Arrange N jobs in ascending order of f (i) and in a favour of the job with the least sum of process times on all M machines.

Step 3: Calculate the makespan of the predetermined schedule through the recursive relation.

$$T^{k}{}_{ij} = Max[T^{k-1}{}_{ij}, T^{k}{}_{i,j-1}] + t_{ij} \quad (2)$$

Where, T^{k}_{ij} is the cumulative processing time up to the kth order for the ith job through j^{th} machine.

2.2 Rajendran Heuristic (CR)

Rajendran (CR) [10] has implemented a heuristic for the flowshop scheduling with multiple objectives of optimizing makespan, total flow time and idle time for machines. This improvement heuristics, the first seed is taken from CDS algorithm. The heuristic preference relation is proposed and is used as the basis to restrict the search for possible improvement in the multiple objectives. This method is simple but repeating of steps is needed. This makes the evaluation time of the heuristic as high.

Step 1: The seed sequence is swapped to yield more sequences. In the first step the first and second jobs are interchanged while other jobs are retained in their same position. In the same way as the second step, the first and third jobs are interchanged. Similarly the sequences are generated and the makespan and flow time for the swapped sequences are calculated.

Step 2: The seed sequence is assumed as S and the swapped sequences are taken as S'.

Step 3: Select the best sequence, among the swapped sequences.

2.3 Baskers Pascal's Triangle Method (PTM)

Baskar and Anthony Xavier [11] reported a dummy machine concept along with Pascal's Triangle Method (PTM) and Johnson's algorithm. The advance of PTM had been proposed by Baskar and Anthony Xavior [12] as a current view.

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \\ . & . \\ . & . \\ T_{n-1,n} & T_{n-1,2} \\ T_{n1} & T_{n2} \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1m} \\ t_{21} & t_{22} & \cdots & t_{2m} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ t_{n-1,1} & t_{n-1,2} & \cdots & t_{n-1,m} \\ t_{n1} & t_{n2} & \cdots & t_{nm} \end{bmatrix}$$

$$\begin{bmatrix} (m-2)C_0 & 0 \\ (m-2)C_1 & (m-2)C_0 \\ . & . \\ . & . \\ (m-2)C_{(m-2)} & (m-2)C_{(m-3)} \\ 0 & (m-2)C_{(m-2)} \end{bmatrix} \quad (3)$$

Pascal's Triangle is the triangular representation of the combinational elements of nC_r,

The PFS problem of 'n' jobs to be processed in 'm' machines will be converted to 'n' jobs , two machine problem using this Heuristic and the problem will be solved for minimum Makespan using Johnson's Algorithm which is a proved one for a two machine PFS problem. Even though it is the lastest method with better results, the steps involves in this heuristics are long process based and huge to solve.

3 PROPOSED HEURISTIC-EXPONENTIAL DISTRIBUTION TECHNIQUE

This algorithm distributes a factor in the processing time of the jobs at each machine from the advancement of classical algorithm. This factor added to the job is evaluated through the exponential equation, which gives a value of the index to the respective job. By sorting the index valve in descending an optimal sequence can be obtained.

From the illustration of Palmer, the Palmer's sequence can provide an optimum elapsed time. The advancement of his methodology is proposed as a new heuristic to evaluate the optimal elapsed time. This heuristic provides 'n' values, and these are to be descended and respective jobs to be sequenced.

Using the taillard benchmark problem [13], the newly proposed heuristic is compared with existing algorithms. The processing times vary from 1 to 99 time units and they are generated using a random number generator for different seeds.

Step 1: Let 'n' number of jobs to be machined on 'm' machines. It is assumed that all jobs are present for processing at time zero. And one job can run on one machine at a time without changing the machine order.

Step 2: The exponential index to be calculated using the exponential equation (4) for n jobs.

$$y_j = \sum_{i=0}^{i=m-1} (2.61 * m - \exp(i)) * T_{m-i} \quad (4)$$

Where, Y_j = exponential index valve for jth job,
m = number of machines
$T_{(m-i)}$ = process time of job under $(m-i)^{th}$ machine

Step 3: Sort the exponential index in descending order.

Step 4: Based on the sorted order, the jobs to be sequenced.

4. ANALYSIS OF EXPONENTIAL DISTRIBUTION TECHNIQUE

The set of results is obtained from the C++ program for this newly proposed heuristic. From Table 1-3, the results are compared and examined to the lower bound values of Taillard's benchmark problems. The results of each set are graphical represented in Fig. 1-3.

The overall percentage nearer to lower bound (LB) is tabulated and graphically represented in Table 4 and Fig. 4 respectively. It shows that the EPDT performs better as compared to existing algorithms and reaches LB about 78.5%.

5. CONCLUSIONS

In this research work, a new heuristic was proposed based on the exponential function from mathematical and computational criteria to identify the optimal makespan giving sequence in a flow shop. The attempt made was good for high number jobs or machine; it was proved by the overall % nearer to LB chart. The processing times of jobs on machines are taken from taillard benchmark problems. It noticed that obtained minimum elapsed time for an optimal sequence from the proposed heuristic and it is compared with existing heuristics. The C++ program was generated for comparing the results computationally. The proposed heuristic give a better optimal sequence about 80%, compared to others algorithms.

Table 1. Comparison of heuristic's makespan for 5 machines, 20 jobs.

Taillard Seeds	Lower Bound	GUPTA	CR	PTM	EPDT
873654221	1232	1,409	1377	1398	1377
379008056	1290	1424	1468	1481	1360
1866992158	1073	1255	1379	1387	1236
216771124	1268	1485	1548	1438	1564
495070989	1198	1367	1387	1354	1342
402959317	1180	1387	1411	1310	1385
1369363414	1226	1403	1381	1438	1268
2021925980	1170	1395	1404	1339	1504
573109518	1206	1360	1425	1428	1434
88325120	1082	1196	1284	1237	1298

Table 2. Comparison of heuristic's makespan for 10 machines, 20 jobs.

Taillard Seeds	Lower Bound	GUPTA	CR	PTM	EPDT
587595453	1448	1829	1887	1896	1915
1401007982	1479	2021	2121	2073	1928
873136276	1407	1773	1786	1883	1737
268827376	1308	1678	1628	1703	1727
1634173168	1325	1781	2693	1718	1713
691823909	1290	1813	1835	1757	1618
73807235	1388	1826	1659	1725	1870
1273398721	1363	2031	1878	1821	1928
2065119309	1472	1831	1851	1832	1832
1672900551	1356	2010	1878	1876	2035

Table 3. Comparison of heuristic's makespan for 20 machines, 20 jobs.

Taillard Seeds	Lower Bound	GUPTA	CR	PTM	EPDT
479340445	1911	2833	2700	2614	2606
268827376	1711	2564	2600	2608	2516
1958948863	1844	2977	2742	2776	2575
918272953	1810	2603	2550	2628	2561
555010963	1899	2733	2815	2799	2513
2010851491	1875	2707	2518	2588	2697
1519833303	1875	2670	2730	2551	2687
1748670931	1880	2523	2582	2568	2676
1923497586	1840	2583	2615	2615	2553
1829909967	1900	2707	2472	2695	2372

Table 4. Overall comparison of heuristics using Taillard benchmark problems.

No. of machines	CR	GUPTA	PTM	EPDT
5	84.80 %	87.20 %	86.38 %	86.86 %
10	73.30 %	74.61 %	75.77 %	75.80 %
20	70.54 %	69.06 %	70.18 %	72.09 %

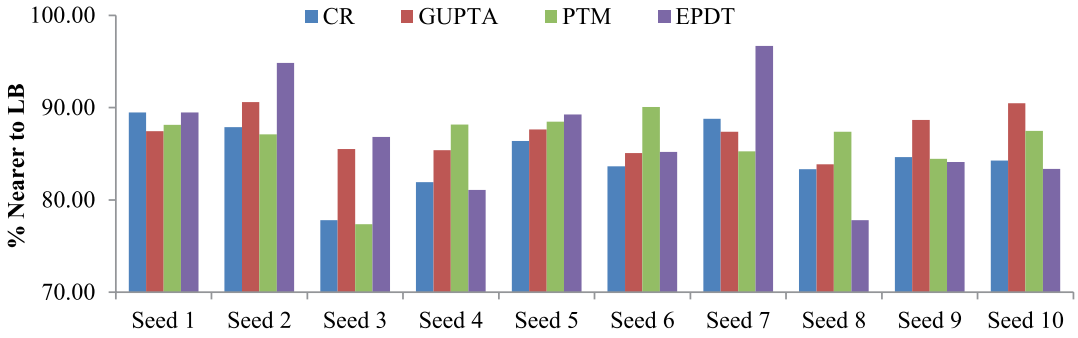


Fig. 1. Comparison of Heuristic's makespan for 5 machines, 20 jobs.a

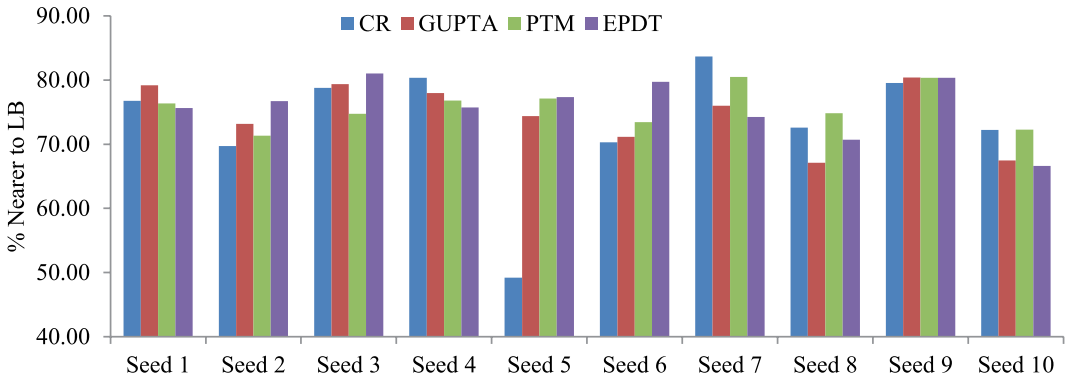


Fig. 2. Comparison of Heuristic's makespan for 10 machines, 20 jobs.

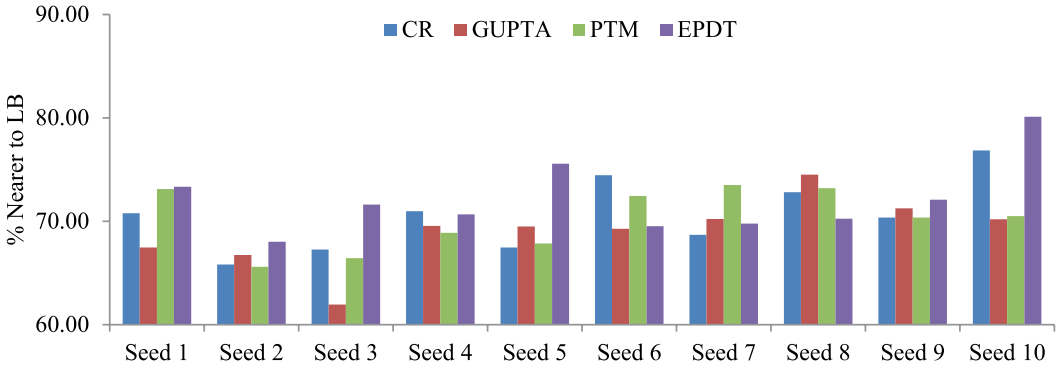


Fig. 3. Comparison of Heuristic's makespan for 20 machines, 20 Jobs

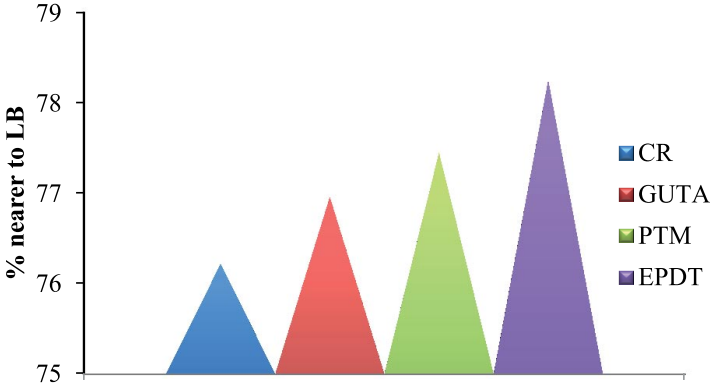


Fig. 4. Overall comparison of Heuristic's using Taillard benchmark problems.

6. REFERENCES

1. Johnson, S.M. Optimal two and three machine production scheduling with setup times included. *Naval Research* 1(1): 61–68 (1954).
2. Ignall, E. & L. Schrage. Application of the branch and bound technique to some flow-shop scheduling problems. *Operations Research* 15: 400 – 412 (1965).
3. Palmer, D.S. Sequencing jobs through a multi-stage process in the minimum total time – a quick method of obtaining a near optimum. *Operations Research* 16: 101-107 (1965).
4. Palmer, K. Sequencing rules and due date assignments in a job shop. *Management Science* 30(9): 1093 – 1104 (1984).
5. Hundal, T.S. & J. Rajgopal. An extension of Palmer heuristic for the flow-shop scheduling problem. *International Journal of Production Research* 26: 1119-1124 (1988).
6. Dannenbring, D.G. An evolution of flow-shop sequencing heuristics. *Management Science* 23: 1174-1182 (1977).
7. King, J.R. & A.S. Spachis. Heuristics for flowshop scheduling. *International Journal of Production Research* 18(3): 345-357 (1980).
8. Blackstone, J.H., D.T. Phillips, & G.L. Hogg. A state of art survey of dispatching rules for manufacturing job shop operations. *International Journal of Production Research* 20: 27–45 (1982).
9. Gupta, J.N.D. A functional heuristic algorithm for the flow shop scheduling problem. *Operational Research* 22: 27−39 (1971).
10. Rajendran, C. Theory and methodology heuristics for scheduling in flow shop with multiple objectives. European *Journal of Operational Research* 82: 540−555 (1995).
11. Baskar, A. & M.A. Xavior. A new Heuristic algorithm using Pascal's triangle to determine more than one sequence having optimal/ near optimal make span in flow shop scheduling problems. *International Journal of Computer Application* 39(5): 9-15, (2012).
12. Baskar, A. & M. Anthony Xavior. Optimization of total material processing time in a manufacturing flow shop environment. *Advanced Materials Research* 622-623: 136-141 (2013).
13. Taillard, E. Bench marks for basic scheduling problems. *European Journal of Operational Research* 64(2): 278-285 (1993).

Proceedings of the Pakistan Academy of Sciences 51 (3): 193–199 (2014)

ISSN: 0377 - 2969 (print), 2306 - 1448 (online)

Pakistan Academy of Sciences

Research Article

An Efficient Algorithm and Its Implementation for Armstrong's Indirect Method of Conversion of Narrowband FM to Wideband FM: A Valuable Tool for Communication System Designers

Syed Waqar Shah[1], Mazhar Sher[2], Talha Khan[2], Tariqullah Jan[1*], Haseeb Zafar[1],Syed Riaz ul Hassnain[2], Gulzar Ahmad[1] and Sahibzada Majid Ashraf[1]

[1]Department of Electrical Engineering, University of Engineering and Technology, Peshawar, Pakistan

[2]Department of Electronics Engineering ,University of Engineering and Technology, Abbottabad Campus, Pakistan

Abstract. A novel algorithm is presented to calculate the most appropriate frequency multipliers for the conversion of NBFM to WBFM. This algorithm makes the communication system designer job easier and calculates most accurately the best possible frequency multiplying factors in least possible time. The carrier frequency and frequency deviation of NBFM signal can be increased by the application of these frequency multipliers. The algorithm is implemented in C++ programming language and is applied to various practical design problems. The best possible results are generated with least possible errors. The algorithm can be used for either frequency Doublers or frequency Triplers or frequency Quintuplers or 1000* frequency or combination of them like frequency Doublers and Triplers, frequency Doublers and Quintuplers, frequency Triplers and frequency Quintuplers, etc.

Keywords: Narrowband frequency modulation, wideband frequency modulation, algorithm, communication system

1. INTRODUCTION

FM signal can be generated by two methods namely: Direct Generation and Indirect Generation. The Direct Generation of FM signal involves voltage-controlled oscillator (VCO) in which message signal is used as a control signal and VCO frequency varies linearly with the modulating signal. The severe concern with Direct Generation method is the poor frequency stability. The drift of the carrier frequency is intolerable in commercial FM broadcastings [1]. The indirect method of FM generation consists of two steps:

1. Generation of NBFM signal from the message signal; and
2. Conversion of NBFM to WBFM using frequency multipliers.

The indirect method is preferred in commercial radio broadcasting due to its frequency stability property [2]. Frequency multipliers are of utmost importance in communication system. They are actually non-linear devices used for generation of FM signals [3] with band pass filter to extract desired frequencies. If x(t) is the input signal and it is provided to a non-linear system so that the output obtained from this system is y(t) where

$$y(t)= b_2[x(t)]^2$$

Let $x(t) = \cos(\omega_c t + k_f\int m(t)dt)$ is a FM signal, then y(t) will consist of a dc component $0.5\ b_2$ and a signal $0.5b_2 \cos(2\omega_c t + 2k_f\int m(t)dt)$. A bandpass filter centered at $2\omega_c$ can be used to extract the FM signal whose frequency is twice as compared to original input signal.

This approach can be extended to get signals at $\omega_c, 2\omega_c, 3\omega_c, 4\omega_c, \ldots, n\omega_c$ with frequency deviations

Received, May 2014; Accepted, August 2014
*Corresponding author: Tariqullah Jan: Email: tariqullahjan@nwfpuet.edu.pk

ranging from Δf, 2Δf, 3Δf,…, nΔf. Thus by the application of frequency multipliers it is possible to increase the carrier frequency and frequency deviations of composite FM signal.

An nth multiplier will increase the carrier frequency to $n\omega_c$ and frequency deviation to nΔf. This concept is used to obtain WBFM from NBFM. The NBFM signal can be approximated by the following equation:

$$\varphi_{NBFM}(t)=A[\cos \omega_c t - k_f\, a(t) \sin \omega_c t]$$

The next process involves the use of frequency multipliers to increase the carrier frequency and frequency deviation. When Δf and fc is multiplied by a multiplying factor of n, the tremendous increase in fc is undesirable and frequency heterodyning operation is used to down shift the carrier frequency [4].

Edwin Armstrong is the pioneer and inventor of Wideband FM systems [5] and his major interest was in the suppression and possible elimination of various atmospheric disturbances [6]. WBFM signals have distinct characteristic that their bandwidth not only depends upon the bandwidth of the message signal f_m but also on the modulation index β. WBFM has modulation index having value β>>1 and by varying the value of this β, we can control the bandwidth of the WBFM .

FM commercial transmitter is shown in Fig. 2, where message/modulating signal is first converted to approximate NBFM signal and then multipliers and frequency converters are employed to obtain the WBFM signal. Frequency multipliers are incorporated to change the carrier frequency, modulation index β and frequency deviation [7]. Crystal oscillator having frequencies ranging from 9 MHz to 11 MHz and frequency mixing circuits are employed to perform the frequency down conversion.

3. ALGORITHM DESCRIPTION

The flow chart of the proposed algorithm for the conversion of NBFM to WBFM is presented in Fig. 3. The algorithm [8] is used to determine the most appropriate frequency multiplying factors with the best possible accuracy and in least possible time. This algorithm is used to determine the multipliers works on the following logic:

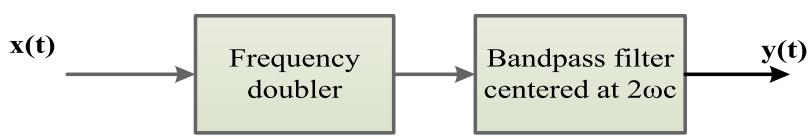


Fig. 1. The block diagram of frequency doubler.

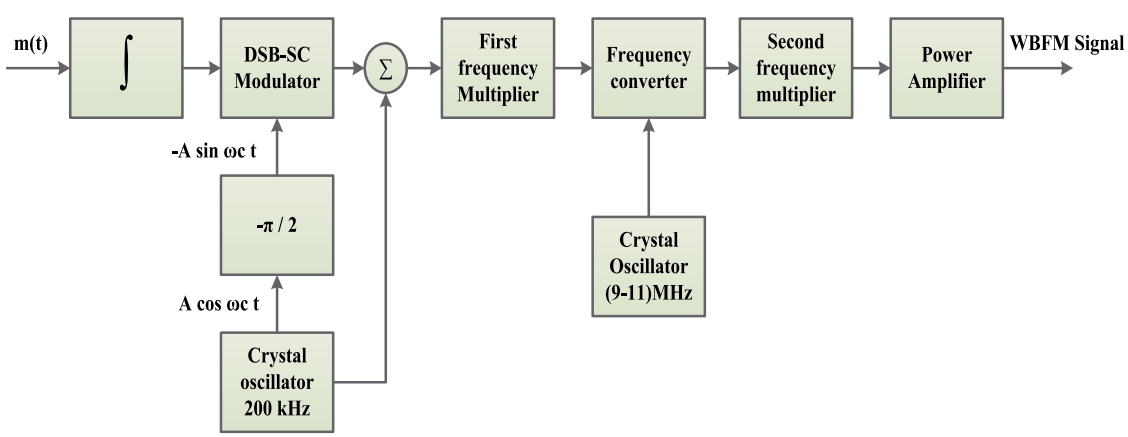


Fig. 2. Block diagram of Armstrong Indirect FM transmitter.

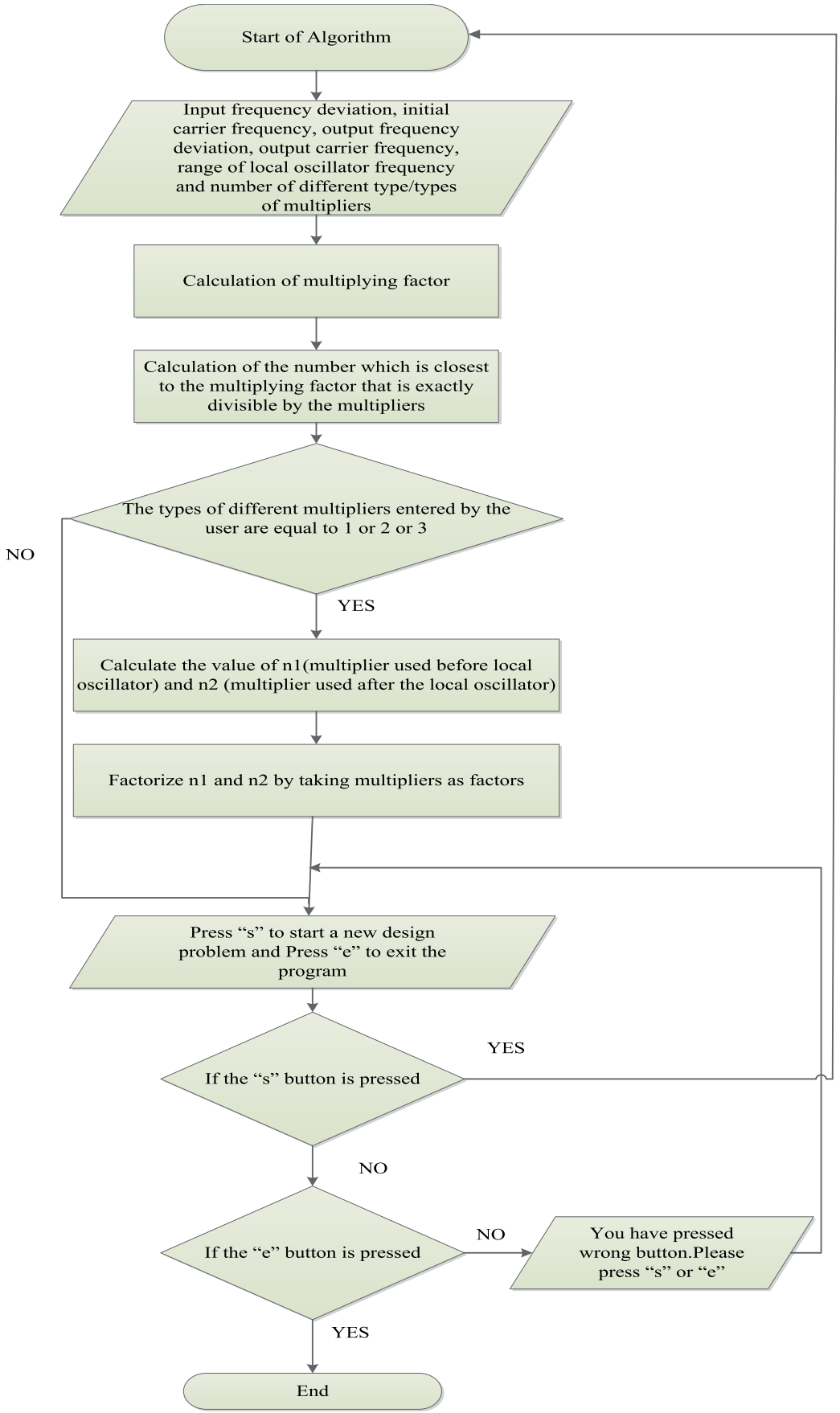


Fig. 3. Flow chart of the proposed algorithm.

To convert a NBFM to WBFM we have to increase the frequency deviation and carrier frequency of the NBFM. For example if we have a NBFM having frequency deviation of 25 Hz(df1) and carrier frequency of 200 kHz(fc1) and we want to change it WBFM having the frequency deviation of 75 kHz(df4) and carrier frequency of 12.8 MHz(fc4).

To perform this task we multiply the frequency deviation and carrier NBFM with a certain number. The multiplication factor is found by dividing the final frequency deviation i.e. 75000 by initial frequency deviation i.e. 25 Hz and we get the number 3000.We have to multiply the NBFM signal with 3000 to change it to WBFM signal.

We do not have any electronic device that can directly multiply the frequency up to some value i.e. 3000 in this case that's why we convert the multiplication factor to some smaller factors so that available electronic devices like doublers, triplers andquintuplers or their combinations can be used to accomplish the required task.

If we utilize only doublers and triplers for the above case to get the multiplication factor of 3000. And the possible combination of doublers and triplers for the number closest to 3000 is 3072 which can be accomplished using 10 doublers and 1 tripler.

The first part of algorithm consists of finding the most suitable multiplying factor in terms of doublers, triplers, quintuplers etc. The algorithm checks whether the number is completely divisible by any combination of the different type of multipliers provided by the user. Let for the above case the program will check whether 3000 is completely divisible by the multiplication of any possible combination of doublers and triplers. If not then it will find the closest possible number completely divisible by the multiplication of any combination of doublers and triplers.

The algorithm determines the closest number by the following logic:

Algorithm copies the number (3000) into two different variables (num1 and num2) and increases the number (3000) by 1 using one variable num1 (3001 in the above case) and then checks that whether that number is completely divisible by the multiplication of any combination of doublers or triplers and in the next step it will decrease that number by 1 using other variable num2 (2999 in the this case) and then checks the same condition. If the condition does not meet it again increases the previously increased number num1 by 1(the number will be 3002) and the checks the condition if it is not satisfied then will decrease the previously decreased number by 1 (the number n_2 will be 2998) and then checks the condition and so on.

In this way a number can be obtained that is closest to original number (3000 in this case) and is completely divisible by the multiplication of the combination of different type of multipliers given (doublers and triplers in this case).

The direct multiplication by the help of doublers and triplers will result in increase of carrier frequency beyond the permissible value. This difficulty can be resolved by incorporating the frequency mixing process.

Initially we multiply both initial frequency deviation and carrier frequency of NBFM by an appropriate combination of multipliers (doublers and triplers in the our case) from the total combination of multipliers and then we will translate the carrier frequency using local oscillator and then again multiply the frequency deviation and carrier frequency with the remaining combination of doublers and triplers.

If we consider the above case, and let we have oscillators available in inventory stock having frequency range from 10 MHz to 11 MHz. We multiply the initial frequency deviation with 64(using six doublers) and the carrier frequency shifts to 12.8 MHz.

By the application of a local oscillator having frequency 10.9 MHz, the carrier frequency is shifted to 1.9 MHz (fc3). Then this frequency fc3 is multiplied by 48 (which is the remaining combination of multipliers having four doublers and one tripler) to obtain the final carrier frequency of 91.2 MHz.

Our proposed algorithm automates the above challenging task of finding the suitable factor of 3072 by using the following logic:

This novel algorithm splits the original number n into two numbers n_1 and n_2. The values of n_1 and n_2 greatly depend on the availability of different oscillators.

First of all, we will use only one type of multiplier and increase its power up to the last permissible power. As in the above example we choose n_1=3072, n_2=1 and check the condition that the resulting carrier frequency is in the range of local oscillator or not. If not then we swap the values assigned to n_1 and n_2 so that the new values are n_1=1 and n_2=3072 and check the range of the carrier frequency.

The next step involves the division of fc_4 by the first type of multiplier (the factor 2 in this example) and check of carrier frequency for the range of available local oscillators, for n_2=2 and n_1=1536 and in case of failure, the condition will be rechecked after swapping n_1 and n_2. After this step the power of the 1st multiplier (doubler in this case) is increased continuously up to its last value and after increasing every time we also swap the numbers. Here for each case, the program also checks the condition that whether resulting carrier frequency fc is in the range of the local oscillator's frequencies. If it does not then we will take n_2 the 2nd multiplier (tripler in the above case) and then n_1=n/n_2. If the condition does not fulfill, then the algorithm will multiply 1st multiplier with n_2 and will take the modified value of n_2 and each time will check the validity of condition and also swaps the numbers (the program swap the numbers every time when they are changed and also checks the condition). Then the power of first multiplier is increased up to its last value (10 in this case). If during this process, the condition does not meet it will make n_2 the square of 2nd multiplier and the multiply it with the 1st multiplier up to its last power and if condition does not satisfy it will make n_2 the cube of 2nd multiplier and will check the condition and in case condition of carrier frequency in permissible range does achieve then multiply it with the 1st multiplier up to its last power and so on (depending on n_2 it is also modifying n_1 by n_1=n/n_2). This process is continued for each possible factor of the resulting number (3072 in this case) by using the multipliers given (by using doublers and triplers in this case).

The algorithm successfully finds the two possible values of n_1 and n_2 which will result in the desired frequency deviation and carrier frequency.

A. Design Problem # 1

The task is to design an Armstrong indirect FM modulator which is capable of generating a wide band FM signal with carrier frequency 97.3 MHz and frequency deviation 10.24 kHz. A NBFM generator having output carrier frequency fc = 20 kHz and frequency deviation Δf = 5 Hz is available in inventory store. To convert this NBFM to WBFM the only multipliers available are doublers. The available local oscillators have adjustable frequency range of 400 kHz to 500 kHz for frequency mixing purposes.

B. Design Problem # 2

Design an FM modulator capable of generating an FM carrier with a carrier frequency of 96 MHz and frequency deviation Δf = 20 kHz. A NBFM generator with output carrier frequency fc = 200 kHz and frequency deviation Δf = 9.7656 Hz is available. A local oscillator with adjustable frequency in the range of 9 to 10 MHz and only frequency doublers are available.

C. Design Problem# 3

Design a commercial FM transmitter using Armstrong indirect method. The final output specifications are following:

1) Carrier frequency= 91.2 MHz
2) Frequency deviation = 75 kHz.
3) A narrowband FM generator with fc = 200 kHz and Δf = 25 Hz is available.
4) Local oscillator with adjustable frequency in the range of 9 to 11 MHz
5) Only frequency doublers and Triplers are available.

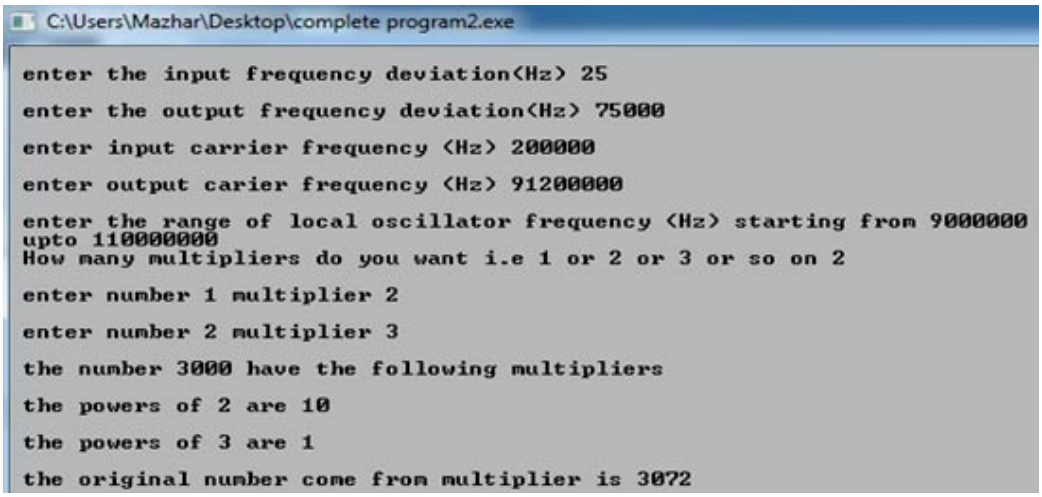


Fig. 4. The screen visual showing the execution of proposed algorithm.

Table 1. The results obtained for various design problems.

Serial #	Input Frequency Deviation	Output Frequency Deviation	Input Carrier Frequency	Output Carrier Frequency	Range of Local oscillator	Multipliers	First Multiplier	Second Multiplier
Design Problem # 1	5	10240	20000	97300000	400000-500000	only doublers	16	128
Design Problem # 2	9.7656	20000	200000	96000000	9000000-10000000	only doublers	64	32
Design Problem # 3	25	75000	200000	91200000	9000000-11000000	Doublers and Triplers	64	48
Design Problem # 4	10	75000	100000	98100000	10000000-11000000	doublers, triplers and 5 times multiplier	124	60
Design Problem # 5	12	22000	200000	99100000	9000000-11000000	Triplers and 5 times multipliers	74	25
Design Problem # 6	30	76000	200000	101000000	7000000-11000000	Doublers and Triplers	54	48
Design Problem # 7	15	75000	200000	10100000	9000000-11000000	Doublers and Triplers	54	96
Design Problem # 8	20	75000	200000	100000000	9000000-11000000	Doublers and Triplers	54	72
Design Problem # 9	18	75000	200000	97000000	9000000-12000000	doublers	64	64
Design Problem # 10	40	75000	200000	106000000	9000000-11000000	Doublers and Triplers	72	27

D. Design Problem# 4

Design an Armstrong indirect FM modulator to generate a WBFM signal having fc = 98.1 MHz and Δf = 75 kHz. A narrowband FM generator with fc=100 kHz and Δf = 10Hz. The local oscillator available in the stock room has an adjustable frequency in the range of 10 to 11 MHz. Any multiplier like doublers, triplers, quintuplers etc can be used.

4. TEST OF PROPOSED ALGORITHM AND RESULTS

The algorithm was implemented in C++ programming language [9,10] and the above mentioned four design problems were tested .The screen shot for one of the problems is presented in Fig. 4.

The overall results are summarized in the form of table 1. The algorithm is also tested for 6

different design problems and the following results were obtained.

5. CONCLUSIONS

The proposed algorithm not only makes the lengthy and tiresome process of conversion of NBFM to WBFM much easier but is very accurate too. The overall accuracy of the algorithm is 97.62%. The task of communication system designer is made easier by implementing this algorithm. This algorithm can be implemented on printed circuit board in near future to compare its performance.

6. REFERENCES

1. Haykin, S. & M. Moher. *An Introduction to Analog and Digital Communications*, *2nd ed*. John Wiley & Sons, New York, USA (2006).
2. Proakis, J. G. & M. Salehi. *Digital Communications, 5th ed*. McGraw Hill, New York, USA (2007).
3. Ramteke, S.D. Design and Development of Digital Frequency Multipliers. *Proceedings of the Second International Conferebce on Computer Applications (ICCA 2012)*. Pondicherry, India, Volume 1, p.75-79 (2012).
4. Lathi, B.P. & Z. Ding. *Modern Digital and Analog Communication System*, *4th ed.* Oxford University Press, Cary, North Carolina,USA (2009).
5. Schwartz, M. Armstrong's invention of noise-supressing FM. *IEEE Communications Magazine*: 20-23 (2009).
6. Frost, G.L. *Early FM Radio: Incremental Technology in Twentieth-Century America.* The Johns Hopkins University Press, Baltimore, Maryland, USA (2010).
7. Thomas, T. G. & S. C. Sekhar. *Communication Theory*. Tata McGraw Hill, New Delhi, India (2005).
8. Sedgewick, R. & K. Wayne. *Algorithms, 4th ed.* Pearson Education Incorporation, Boston, Massachusetts, USA (2011).
9. Weiss, M.A. *Data Structures & Algorithm Analysis in C++, 4th ed.* Prentice Hall, Englewood Cliffs, NJ, USA (2013).
10. Stroustrup, B. *The C++ Programming Language, 4th ed.* Pearson Education Incorporation, Boston, Massachusetts,USA (2013).

Proceedings of the Pakistan Academy of Sciences 51 (3): 201–208 (2014)

ISSN: 0377 - 2969 (print), 2306 - 1448 (online)

Pakistan Academy of Sciences

Research Article

Performance of Six-Pulse Line-Commutated Converter in DC Motor Drive Application

Syed Riaz ul Hassnain[1], Haider Zaman[1], Zunaib Ali[1], Muqadsa Iftikhar[1], and Tariqullah Jan[2]*

[1]University of Engineering & Technology Peshawar, Abbottabad Campus, Pakistan
[2]University of Engineering & Technology, Peshawar, Pakistan

Abstract: This paper presents the speed control of DC motor using six pulse controlled rectifier. The conventional Proportional Integral (PI) control is used for firing angle control. The armature current is fed back and compared with reference current representing desired speed values. The proposed system is simulated using SimPowerSystem and Control System Matlab toolbox. The time domain plot of reference and actual armature current are shown in results section. The results are satisfactory with deleterious effect on input current. The frequency plot of input current is provided to show the harmonic contents, generated as a result of control operation.

Keywords: Silicon control rectifier (SCR), DC motor, proportional integral control, harmonics, displacement factor and distortion factor, line-commutated converter (LCC)

1. INTRODUCTION

The continuous popularity of DC motors is due to their efficient performance and control advantages over ac motors. The scope of DC motor varies from high speed automation to electric vehicles. The dc motors are increasingly used in applications where speed or torque needs to be varied and controlled with high accuracy. The most commonly used DC motors for variable speed are series and separately excited [1], but generally DC series motor is used for traction purposes.

With the development of high power solid state switches, the DC motor control applications started to grow almost exponentially. Today, broadly two methods are employed (based on switches controllability) to set the applied voltage as a variable parameter. In the first method applied voltage can be made variable by using controlled rectifiers which provide variable DC from a fixed ac shown in Fig. 1(a) whereas in the second method uncontrolled rectifiers are used to produce a fixed DC from a fixed ac and at the output of rectifier a DC chopper circuit is used which provide variable DC just by changing the duty cycle shown in Fig. 1(b) [2].

In this paper our major focus is to discuss the use of SCR based converters for speed regulation i.e. to get variable DC voltage from a fixed ac voltage. The use of SCR allows simple voltage variability just by varying the firing angle and thereby the speed of the motor. For the speed regulation of DC motor, the speed needs to be monitored as the speed of DC motor changes with load torque [3]. The control circuit corrects the firing angle of SCR, which in-turn changes the armature voltage, thereby regulating the speed. The control circuit used here consists of PI current controller, which takes current as feedback, compares it to the reference and then generates firing angle of SCRs to minimize the error and regulate the speed [4-5]. The benefits of feedback network (which is also called closed loop control system) are of high level of accuracy, compensation of load variations and system nonlinearities, fast response and continuous monitoring of process [6]. The block diagram of DC motor fed with closed loop controlled rectifier is shown on Fig. 2.

First, the mathematical model of DC motor is developed in section 2. Working principle of six

Received, July 2014; Accepted, August 2014
*Corresponding Author: Tariqullah Jan, Email: tariqullahjan@nwfpuet.edu.pk

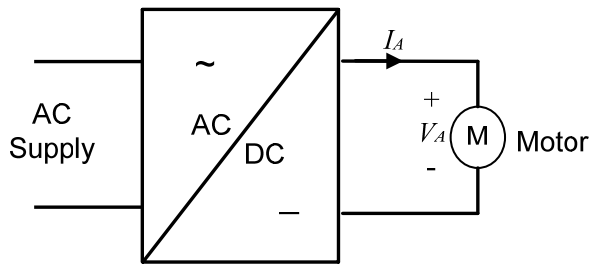


Fig. 1(a). Controlled rectifier fed motor.

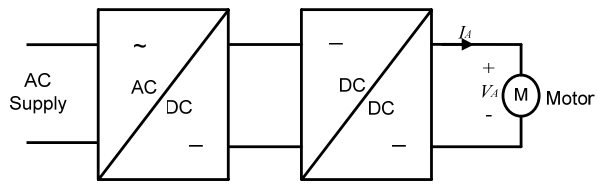


Fig. 1(b). Uncontrolled rectifier followed by DC-DC converter.

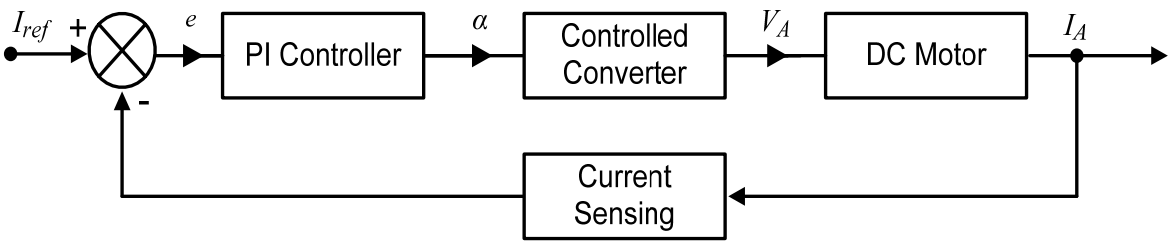


Fig. 2. Block diagram of DC motor fed with closed loop controlled rectifier.

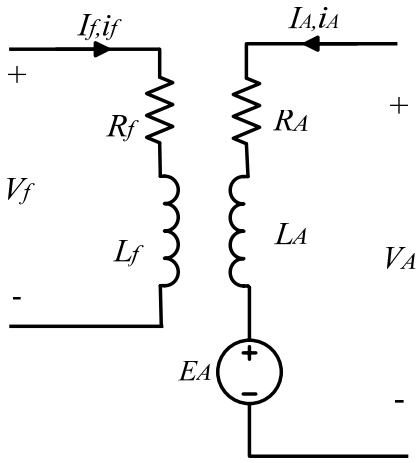


Fig. 3. Separetly excited DC motor.

pulse line-commutated converter is described in section 3. The section 4 presents the complete system responses under a transient condition, followed by a conclusion.

2. DC MOTOR MODELING

The equivalent circuit of commonly used DC motor i.e. separately excited DC motor is shown in Fig. 3. In case of separately excited DC motor the armature circuit and field circuit is independent of each other. When the armature current i_A is passed through the armature circuit placed in a magnetic field produced by field current $i_f$passing through the field circuit, the motor produces a back electromotive force E_A and induced torque τ_{ind} , which balance the load torque at a particular speed. For separately excited DC motor, the current passing through the field windings is independent of the current passing through armature windings. The transient and steady state analysis of separately excited DC motor is derived using Fig. 3.

Transient Analysis

The instantaneous field current is given by differential equation:

$$V_f = i_f R_f + L_f \frac{di_f}{dt} \tag{1}$$

The instantaneous armature current can be calculated from:

$$V_A = i_A R_A + L_A \frac{di_A}{dt} + E_A \tag{2}$$

Where E_A is back emf of the motor and is given by

$$E_A K\omega i_f \tag{3}$$

The torque induced by the motor τ_{ind} is:

$$\tau_{ind} = K i_f i_A \tag{4}$$

The induced torque in terms of load torque is:

$$\tau_{ind} = J\frac{d\omega}{dt} + B\omega + \tau_{load} \tag{5}$$

Where,

$\omega =$ angular speed (rad/s)

$B =$ constant of viscous friction $\left(\mathrm{N}.\frac{\mathrm{m}}{\mathrm{rad/s}}\right)$

$K =$ voltage constant (V/A − rad/s)

$L_A =$ armature circuit inductance (H)

$L_f =$ field circuit inductance (H)

$R_A =$ armature circuit resistance (Ω)

$R_f =$ field circuit resistance (Ω)

$\tau_{load} =$ load torque (N. m)

$J =$ Moment of inertia

Steady-state Analysis

For the steady state analysis all the derivatives with respect to time are put to zero. The resulting steady state average quantities are:

$$V_f = I_f R_f \tag{6}$$

$$\begin{aligned} V_A &= I_A R_A + E_A \\ &= I_A R_A + K\omega I_f \end{aligned} \tag{7}$$

$$\begin{aligned} \tau_{ind} &= K I_f I_A \\ &= \mathrm{B}\omega + \tau_{load} \end{aligned} \tag{8}$$

The power induced or developed is:

$$P_{ind} = \tau_{ind}\omega \tag{9}$$

In modeling of DC motor, the main concern is to find the relationship between speed of DC motor and armature voltage. The speed of the separately excited DC motor in terms of armature voltage can be derived using $Eq.(7)$, i.e.:

$$\omega = \frac{V_A - I_A R_A}{K I_f} \tag{10}$$

Using $Eq.(6)$,$I_f = V_f/R_f$

$$\omega = \frac{V_A - I_A R_A}{K V_f/R_f} \tag{11}$$

From $Eq.(11)$ it is cleared that the speed of motor is directly related to armature voltage V_A, larger the armature voltage greater will be the speed and vice versa[6]. The type of control in which the controlling parameter is voltage, is called voltage control. In practice, the armature and filed currents are kept constant to fulfill the demand of torque while armature voltage is made variable to control the speed. So, in this study a DC motor is fed with the output of a three phase SCR bridge.

3. WORKING PRINCIPLE OF SIX PULSE CONVERTER

The circuit diagram for 3-phase six pulse controlled rectifier is shown in the Fig. 4.The circuit consist of six thyristors, i.e.,

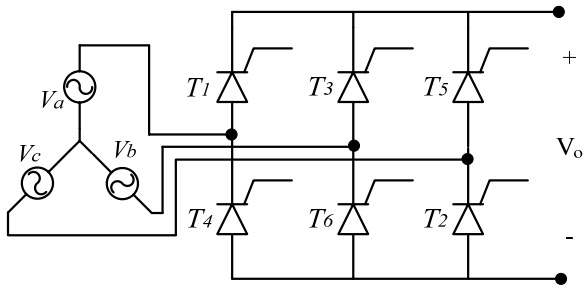


Fig. 4. Six pulse controlled rectifier.

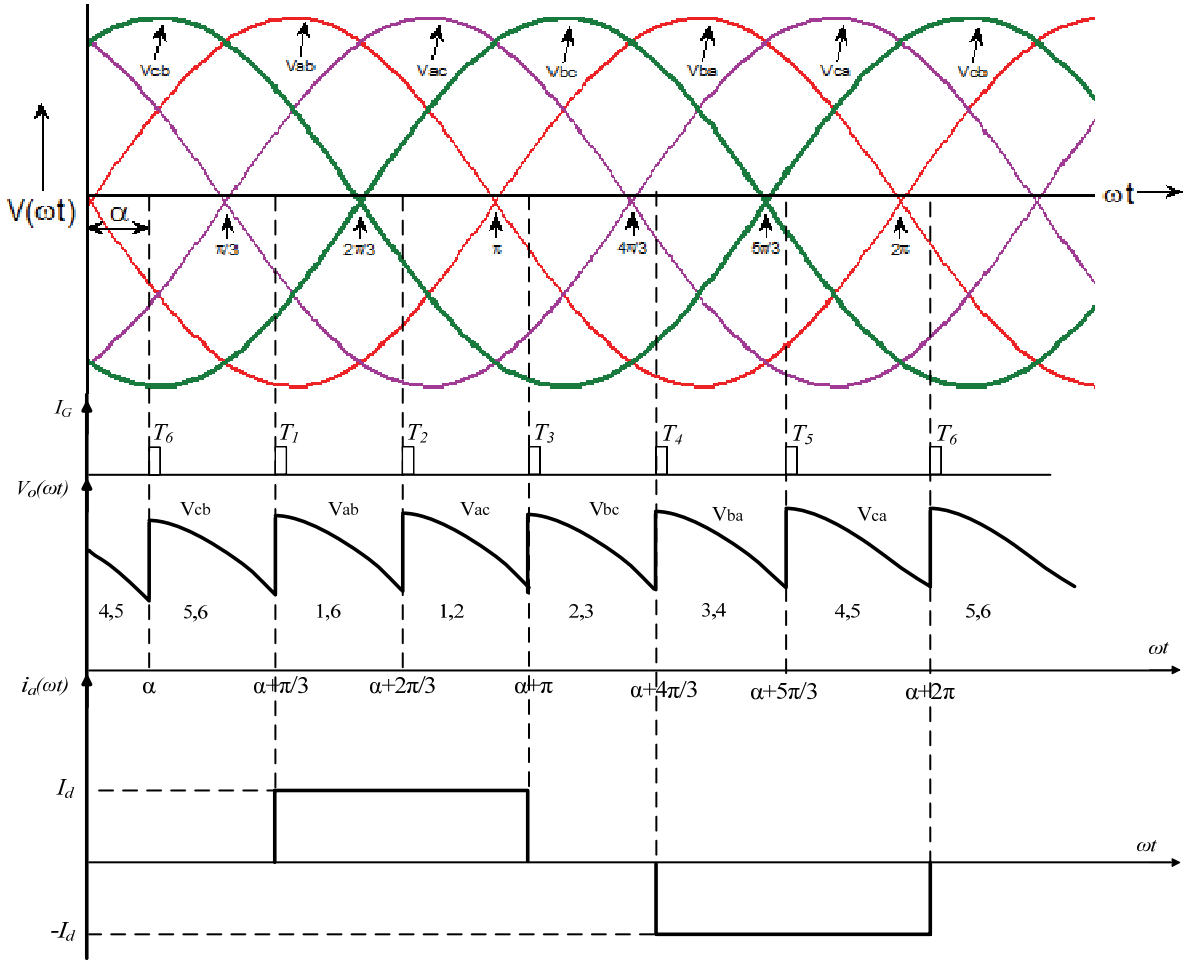


Fig. 5. The $3\emptyset$ line to line voltage waveform $\boldsymbol{V(\omega t)}$, converter output voltage waveform $\boldsymbol{V_o(\omega t)}$, phase "$\boldsymbol{a}$" current waveform $\boldsymbol{i_a(\omega t)}$.

T_1,T_2,T_3,T_4,$T_5$and T_6 for rectification purpose. Three phase system consist of three sinusoidal input voltages (V_a, V_b&V_c) with same frequency and magnitudes i.e. ($f_a = f_b = f_c = f$) & ($V_{ma} = V_{mb} = V_{mc} = V_m$) but shifted 120° from each other. The operating principle of the circuit is that, the pair of SCR connected between the lines havinghighest amount of line-to-line voltage will conduct provided that the gate signal is applied to SCRs at that instant. i.e. SCR needs gate signal in addition to $V_{AK} > 0$. The input and output voltage wavefroms and input current waveform for phase "a" are shown in Fig. 5.

Between $0 \leq \omega t \leq {}^{\pi}/_{3}$ the highest line-to-line voltage is V_{cb}, with T_4& T_5 intially conducting. By firing T_6 at delay angle of α,results V_{cb}at load. It should be noted is that V_{cb}apperas across load till ${}^{\pi}/_{3} + \alpha$. Although beyond the point ${}^{\pi}/_{3}$ the line-to-line voltage V_{ab} has the highest value but until T_1 is not fired, it will not appear across the load and the maximum voltage across load is still V_{cb}.As T_1 is fired, T_5 turns off and now current passes through T_1& T_6 i.e. V_{ab} appears across the load. Another impotant conclusion is that, only one SCR needed to be fired at a time except for the first cycle. Because first SCR of the pair already conducts due to phase sequence.

From Fig. 5 it can be seen that frequency of output pulsating voltage is $6f$ and in this way the harmonic components are shifted to higher frequencyAs compare to three pulse converter, it reduces the need of filtering at the output[7-8].The output DC voltage of a controlled rectifier is a function of the firing angle α and it can be calculated from Fig. 5. The relation is:

$$V_{dc} = \frac{1}{{}^{\pi}/_{3}} \int_{\alpha+{}^{\pi}/_{3}}^{\alpha+{}^{2\pi}/_{3}} \sqrt{3}V_m \sin(\omega t)\, d(\omega t) \qquad (12)$$

$$V_{dc} = \frac{3\sqrt{3}V_m}{\pi} \int_{\alpha+{}^{\pi}/_{3}}^{\alpha+{}^{2\pi}/_{3}} \sin(\omega t)\, d(\omega t)$$

$$V_{dc} = \frac{3\sqrt{3}V_m}{\pi} -\cos(\omega t) \Big|_{\alpha+{}^{\pi}/_{3}}^{\alpha+{}^{2\pi}/_{3}}$$

$$V_{dc} = \frac{-3\sqrt{3}V_m}{\pi}\left[\cos\left(\alpha + {}^{2\pi}/_{3}\right) - \cos\left(\alpha + {}^{\pi}/_{3}\right)\right]$$

$$V_{dc} = \frac{3\sqrt{3}V_m}{\pi}\cos(\alpha) \qquad (13)$$

$$V_{dc} = V_{dm}\cos(\alpha) \qquad (14)$$

$$V_n = \frac{V_{dc}}{V_{dm}}\cos(\alpha) \qquad (15)$$

Where V_{dm} is the average output voltage at$\alpha = 0$and V_n is normanilzed average voltage.The plot in Fig. 6 shows the normalized output DC voltage verses the firing angle α.

It can be seen from Fig. 5 that for $\alpha = {}^{\pi}/_{3}$ the rectified output voltage reaches zero crossing. If α is increased beyond ${}^{\pi}/_{3}$ i.e. $\alpha > {}^{\pi}/_{3}$, the load voltage becomses discontinous for resistive load where as for inductive load the negative voltage appears across load. Fig. 6 shows that by varying α between $0\ to\ {}^{\pi}/_{2}$ output varies between 1 & 0 i.e. rectification region and by varying α between ${}^{\pi}/_{2}\ to\ \pi$ output varies between 0 & -1 i.e. inversion region. Rectification region is represented by 1^{st}Quardant and inversion region by 4^{th}Quardant resulting in 2 Quardant operation.

There is also an issue in using six pulse converter i.e. the effect of converter on input power factor. The power factor for highly inductive load (motor) can be calculated as:

$$P.F = \frac{P_{av}}{3\, V_{irms}\, I_{irms}}$$

$$P.F = \frac{6\, V_m\, I_o}{3\, I_o \pi \cdot V_m\sqrt{2}}\cos(\alpha)$$

$$P.F = 0.9\ \cos(\alpha) \qquad (16)$$

From $Eq.(17)$ it can be seen that by varying α between $0\ to \pi$, the power factor also vary between 0.9 & 0for rectification region and 0 & $-$ 0.9 for inversion region.

4. RESULTS AND DISCUSSION

The circuit consist of sixpulse line-commutated converter (LCC), DC motor and the control system; consisting of PI controller is simulated using Simulink. The simulated system is shown in the Fig. 7. The Synchronized 6-Pulse Generator

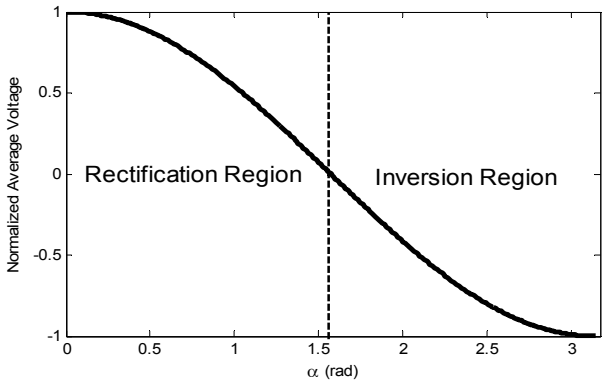


Fig. 6. Normalized average voltage as function of α.

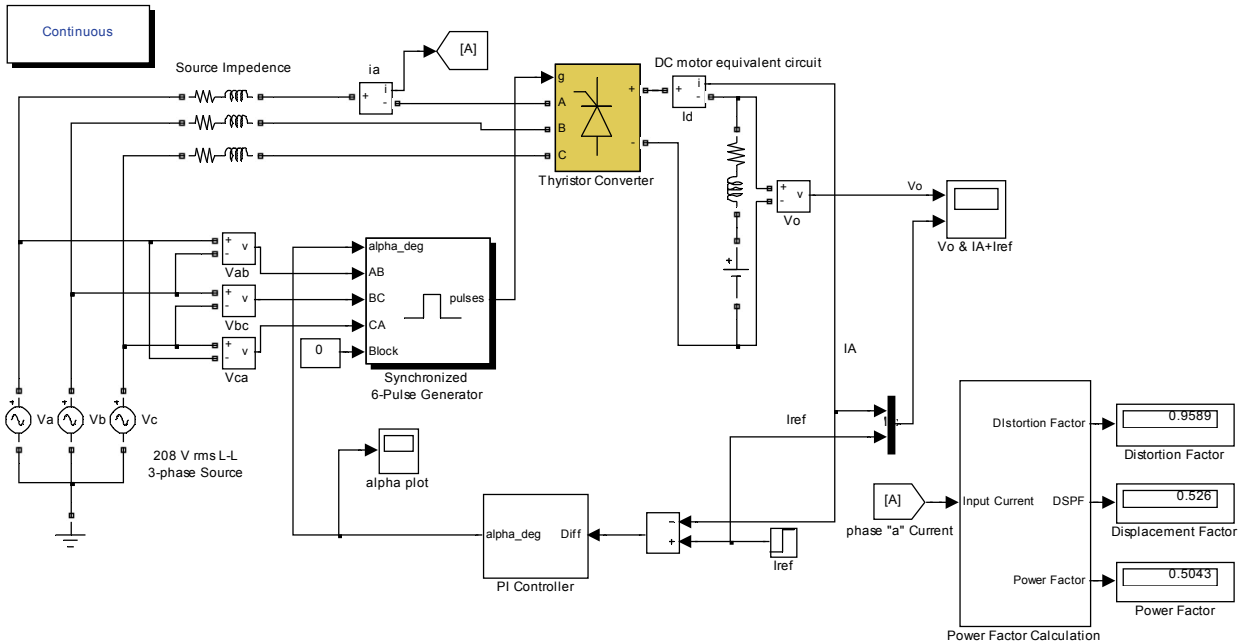


Fig. 7. Model for speed control of DC motor.

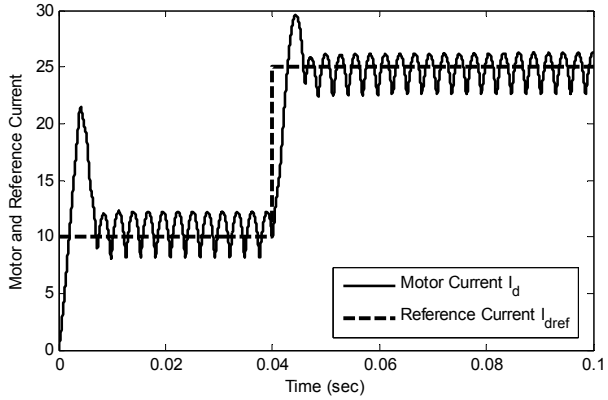


Fig. 8. Variation of Motor Current Corresponding to Reference Current.

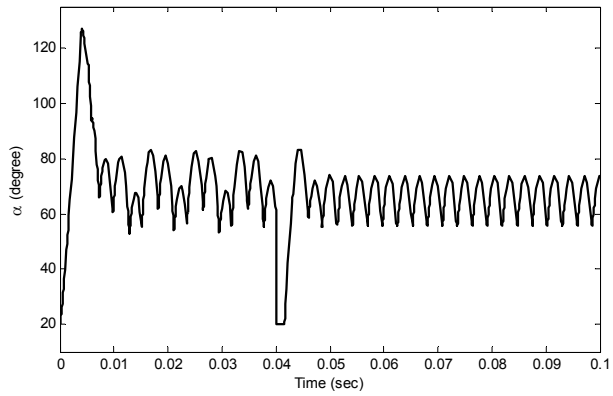


Fig. 9. Variation in $\boldsymbol{\alpha}$ to achieve desired Reference Current.

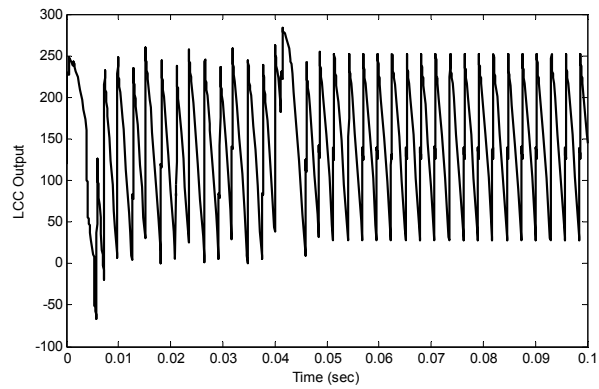


Fig. 10. Variation inconverter output to achieve desired Reference Current**.**

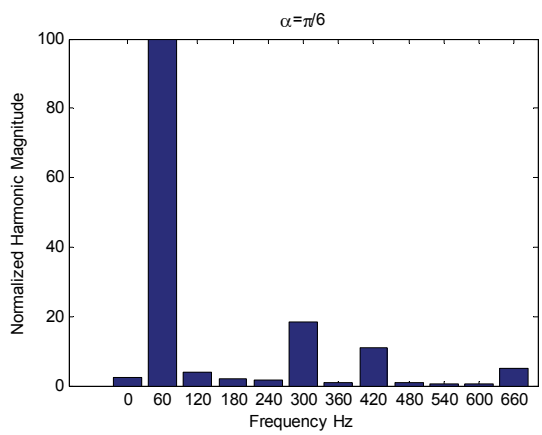


Fig. 11. Frequency Specturm for $\boldsymbol{\alpha} = \boldsymbol{\pi}/\mathbf{6}$.

block is used here to fire the six thyristors (Thyristor Bridge) of a six-pulse converter. The output of the block is a vector of six pulses individually synchronized on the six thyristor voltages. The DC motor is modelled as series connected RL and a DC source, where the DC source representing the back emf.

In this paper the speed is controlled by conrolling the current through the motor. For a desired value of speed, the reference current is provided and using PI controlled the reference current is achieved in a little time. Thetime domain plot for variation of motor armature current I_A for corresponding reference current I_{ref} is shown in Fig. 8. The pattern of firing angle α and LCC output voltage for desired I_{ref} is shown in Fig. 9 and Fig. 10, respectively. Fig. 8 shows that intially, the current through the motor was zero but when the value of reference current is set as "10", the motor current started increasing from zero level and reached the same level as reference current within microseconds. As the reference current is achieved, current through the motor remained constant for the rest of time. Again, when after sometime the reference current is varied from $10\ to\ 25$, the motor achieved and maintained the same current, which implies that the current is regulated. The current regulation directly implies that the speed is also regulated.

One of the important issue needed to be analyzed, is the effect of converter on input power factor because according to $Eq.(17)$ the power factor is the function of firing angle α. The power factor block in Fig. 7 is used to compute the power factor . Power factor is product of displacement factor(due to phase difference between voltage and fundamental component of input current) and distortion factor(due to distortion in input current). For different values of α, the value of distortion and displacement factor is listed in Table.1. For $\alpha = {}^{\pi}/_{6}$ specturm of input current is shown in Fig. 11.

Table 1. Six pulse converter power factor, distortion factor and displacement factor for different α's.

α	Displacement Factor	Distortion Factor	Power Factor
${}^{\pi}/_{6}$	0.758	0.975	0.739
${}^{\pi}/_{4}$	0.639	0.967	0.618
${}^{\pi}/_{3}$	0.487	0.955	0.465

5. CONCLUSIONS

The line-commutated converter with conventional PI control was used for speed regulation of DC motor. The system was tested for the step change in reference current and it provided satisfactory results. When the load torque increases due to extra load i.e.$\tau_{load} > \tau_{ind}$, the motor speed ultimately decreases, this will increase the error "e". To minimize the error the PI controller changes the firing angle of SCR to increase the voltage V_A applied to the armature of DC motor, which increases the armature current I_A, thereby increasing the speed of DC motor. The problem arises at low speed values (i.e. large values ofα) as displacement factor reduces by keeping α large.

6. REFRENCES

1. Kushwah, R. & S. Wadhwani. Speed control of separately excited DC motor using fuzzy logic controller. *International Journal of Engineering Trends and Technology* 4(6): 2518-2523 (2013).
2. Afrasiabi, N. & M.H. Yazdi. DC motor control using chopper. *Global Journal of Science, Engineering and Technology* 8: 67-73 (2013).
3. Shastri, S. & P. Pandey. A comparative analysis of firing angle based speed control scheme of DC motor. *International Journal of Engineering Research and Applications* 3(4): 232-235 (2013).
4. Jaiswal, M. & M. Phadnis. Speed control of DC motor using genetic algorithm based PID controller. *International Journal of Advanced Research in Computer Science and Software Engineering* 3(7): 247-253 (2013).
5. Pal, A.K. & R.K. Mudi. Speed control of DC motor using relay feedback tuned PI, fuzzy PI and self-tuned fuzzy pi controller. *Control Theory and Informatics* 2(1): 24-32 (2012).
6. Rashid, M.H. Multilevel inverters. In: *Alice Dworkin, and Dona King. Power Electronics Circuits, Devices and Applications, 3rd ed.* Pearson Education, p. 409-419 (2003).
7. Pyakuryal, S. & M.A. Matin, Feedback controller for a 3-phase 6-pulse rectifier. *The International Journal of Engineering and Science* 2(8): 23-27 (2013).
8. Pyakuryal, S. & M. Matin. Harmonic analysis for a 6-pulse rectifier. *International Organization of Scientific Research Journal of Engineering* 3 (3): 57-60 (2013).
9. Nedeljkovic, M. & Z. Stojiljkovic. Fast current control for thyristor rectifiers. *IEE Proceedings-Electric Power Applications* 15(6): 636- 638 (2003).
10. Gupta, R., R. Lamba & S. Padhee. Thyristor based speed control techniques of DC motor: A Comparative Analysis. *International Journal of Scientific and Research Publications* 2(6): 1-6 (2012).

Proceedings of the Pakistan Academy of Sciences 51 (3): 209–213 (2014)

ISSN: 0377 - 2969 (print), 2306 - 1448 (online)

Pakistan Academy of Sciences

Research Article

The Culture Performance of 17-α-methyltestosterone Treated Tilapia (*Oreochromis niloticus*) in Fertilized Ponds

Muhammad Hafeez-ur-Rehman[1*], Iftikhar Ahmed[2], Muhammad Ashraf[1], Farzana Abbas[1] and Khalid Javed Iqbal[1]

[1]Department of Fisheries & Aquaculture, University of Veterinary & Animal Sciences, Lahore, Pakistan
[2]Department of Zoology and Fisheries, University of Agriculture, Faisalabad, Pakistan

Abstract: Tilapia is the leading farmed species in all over the world due to its taste and the consumer preference. One of the main constraints in Tilapia culture is its early maturation. The objectives of this study were to determine the effect of different doses of 17-α-methyltestosterone on the growth of *Oreochromis niloticus*. For this study, the experiment expanded over a period of 166 days in four earthen ponds. Fertilization of all the ponds was done with cowdung at the rate of 0.1g N/100 g wet fish body weight daily. The supplementary feed (rice polish) was given at the rate of 2% fish body weight daily. One pond was designated as control pond in which Tilapia was added which was not sex reversed by 17-α-methyltestosterone. In the treated ponds, 17-α-methyltestosterone was applied at the rate of 50, 70, and 90 mg/kg feed. The net fish production was calculated as 850.48, 1026.94, 1117.92 and 1277.17 kg/ha/year with 0, 50, 70, and 90 mg 17-α-methyltestosterone/kg feed pond, respectively. The overall results indicated that the sex-reversed *Oreochromis niloticus* showed markedly increased growth than in control ponds. However, maximum fish growth and production were observed with 90 mg/kg 17-α-methyltestosterone while minimum decrease in fish production was recorded with 50 mg/kg 17-α-methyltestosterone. The Tilapia which was produced through sex reversal had a capability to grow more as compared to normal Tilapia.

Keywords: Culture performance, Tilapia, sex reversed, 17-α-methyltestosterone

1. INTRODUCTION

Tilapias have become one of the most abundantly produced fish in aquaculture, it is now recognized as the "new white fish" accordingly, Tilapias rank as the second fish being cultured solely for food in the world, after carp. Tilapia, *Oreochromis mossambicus* was introduced in Pakistan during 1950, while *Oreochromis aureus* and *Oreochromis niloticus* were imported in 1985 for aquaculture in saline waters of the country [1]. However, they have become nuisance for fish farmers and are considered as pest, due to which although Tilapia is the most popular fish and widely cultured throughout the world but has many constraints of its culture in Pakistan. The constraints are lack of control on its prolific breeding and non-availability of quality fish seed.

Pakistan has vast areas of salt waters which can be best utilized for culturing tilapia species, as this fish is very hardy and quite suitable for these environments. They are also disease resistant, reproduce easily, feed efficiently and can tolerate poor water condition. The production rate of tilapia is 85000 tons per year in Thailand, Taiwan, China, Philippines, Belgium and USA [2]. Methyltestosterone is the most commonly used androgen to direct the sex of tilapia. Sex reversal by oral administration of feed incorporated with methyltestosterone is probably the most effective and practical method for the production of all

Received, April 2012; Accepted, July 2014
*Corresponding Author: Muhammad Hafeez-ur-Rehman; Email: mhafeezurehman@uvas.edu.pk

male Tilapia. Various protocols regarding dose rate and treatment duration have been evaluated. All depend on hormonal treatment with sexually undifferentiated fry. When fish are treated from the beginning to end of the gonadal differentiation period with a proper dose of androgen the resultant fish population will be highly skewed to males. The use of hormones to alter the sex ratios of fish was first demonstrated in species other than Tilapia [3].

Sex reversed tilapia fry production through administration of androgen (17-α-methyltestosterone) is considered to be the efficient and economically feasible method for obtaining all male Tilapia populations [4]. In a previous study, different doses of 17-α-methyltestosterone hormone (MT) used as a growth promoter was administrated to Nile tilapia; *Oreochromis niloticus* in fishmeal based pelleted diet for 90 days. The applied doses were 0.5, 1.0, 2.5, 5, 10, 20 and 40 mg MT/kg feed. The obtained results showed that only the dose of 5 mg/kg was the optimum effective dose in promoting significant final weight, weight gain and SGR of Nile tilapia [5]. Dan and Little [6] compared the culture performance of different species of stains of O*reochromis niloticus* found that, methyltestosterone treatment resulted a final size of fish 10.7 % larger than the mixed sex fish. Hence, this investigation aimed to evaluate culture performance of Tilapia (*Oreochromis niloticus*) treated with 17-α-methyltestosterone in fertilized earthen ponds.

2. MATERIALS AND METHODS

The experiment was conducted over a period of 166 days during the month of May to October in four earthen ponds, each measuring 25m x 8m x 1.5m at the Fisheries Research Farms, Department of Zoology and Fisheries, University of Agriculture, Faisalabad, Pakistan. Fertilization of all the ponds was done with cow dung at the rate of 0.1 g N/100 g wet fish body weight daily [7]. 130 Tilapia were stocked in each pond. The supplementary feed (Rice polish) was given at the rate of 2% fish body weight daily. One pond was kept as control in which Tilapia was added which was not sex-reversed by 17-α-methyltestosterone while in pond 1, pond 2 and pond 3, 17-α-methyltestosterone were applied at the rate of 50, 70, and 90 mg/kg feed, respectively. Fish growth was measured in terms of increase in body weight by random capturing of 20 fish samples from each pond on each fortnight. After obtaining the data, the fish were released back into their respective ponds. The data of growth parameters was subjected to statistical analysis through microcomputer using MSTATC packages following [8].

3. RESULTS

After 166 days of rearing, all Tilapia were harvested from all the ponds. Survival rate for all the fish species was found to be 100% throughout the experimental period. The initial average body weights of Tilapia were 20.2, 22.1, 22.3 and 24.0g while the final were 79.0, 93.1, 103.6 and 112.3 g in control, 50, 70, and 90 mg 17-α-methyltestosterone/kg feed, respectively. There were net gains of 58.8, 71.0, 81.3 and 88.3g. The gross fish production was found to be 1142.65, 1346.59, 1498.46 and 1624.30 kg/ha/year while the net production was 850.48, 1026.94, 1179.2 and 1277.17 kg/ha/year in control, 50, 70, and 90 mg 17-α-methyltestosterone/kg feed, respectively (Fig. 1). However, pond-3 (which was treated with 90 mg/kg feed 17-α-methyltestosterone) showed the best in overall weight gain. Fish body weight varied significantly with treatments as well as over the fortnights ($P \leq 0.05$) The water was taken from the tube well of the Fisheries Research Farms. Mean pH ranged from 7.5-8.8 were within the suitable values for fish culture [9]. Dissolved oxygen (DO) concentrations ranged from 4.73-8.5 mg L^{-1} in the morning and 5.03-7.2 mg L^{-1} in the afternoon. Temperature was 23.9-28.1°C (morning) and 26.9-29.2 ˚C (afternoon). According to Phelps and Popma [10] DO concentrations should remain above 4 mg L^{-1} and the optimum temperature between 26–28 ˚C for ideal fish culture system.

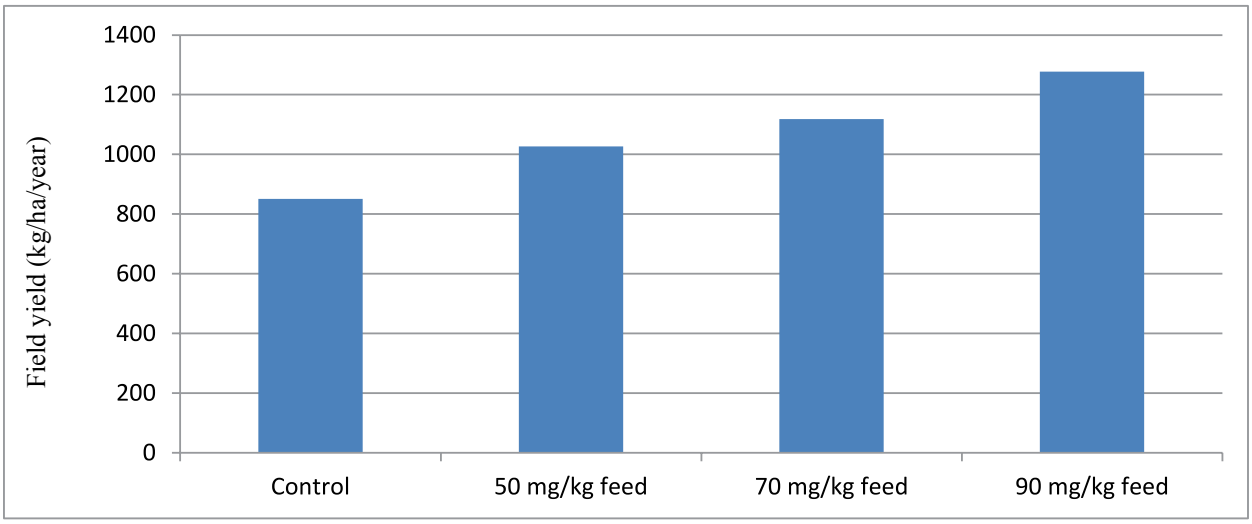


Fig. 1. Net fish production kg/ha/year of tilapia with various dosage of 17-α-methyltestosterone in the fish ponds.

4. DISCUSSION

The results of the present research showed that different dose rates of methyltestosterone affected the growth of *Oreochromis niloticus* significantly ($P<0.05$). All the treatments which received methyltestosterone, showed more average body weight and gain in body weight of *Oreochromis niloticus* than the control (Table 1). This was due to 17-α-methyltestosterone, which was orally administrated in their feed. The same results were observed by Ridha and Lone [11] who observed that treated groups with 17-α-methyltestosterone showed significance increases in weight than control pond. These results are in line with the findings regarding anabolic effect of 17-α-methyltestosterone in fish and all male culture of tilapia by different authors. Hanson et al [12]reported that 10-60 ppm methyltestosterone treatment showed the best growth than control. Varadaraj et al [13] observed faster growth in *O. mossambicus* when fed 17-α-methyltestosterone. These results are also in line with Dan and Little [6] who compared the culture performance of different strains of *O. niloticus* and found that considering all strains, methyltestosterone treatment resulted in a final size of fish 10.7 % larger than mixed sex fish. In the present experiment, it was observed that in treated pond, the maximum growth was observed in pond 3 in which 17-α-methyltestosterone given at the rate of 90 mg than 50, and 70 mg of 17-α-methyltestosterone and this was in accordance with Carvalho and Foresti [14] who gave the different level of treatment Tilapia with 30, 50 or 100 mg of 17-α-methyltestosterone. The highest growth rate was recorded in pond, which received 100 mg of 17-α-methyltestosterone than other treated pond. Semi-intensive production in ponds using fertilizers and supplementary feeding (rice polish) is a mean of producing low cost tilapia in developing countries like Pakistan. The advantages of such culture are widely recognized for rural food supply. It can provide an opportunity to balance the use of supplementary feeding in correlation with the natural food availability and hence reduce the production cost. The reduced growth in control pond which was not sex reversed may be due further propagation of tilapia resulted in higher densities in the system leading to competition for food and space. These results are in line with David et al [15] who reported that limitations of feed availability in the fertilized, semi-intensive system may have exacerbated the relatively poorer growth of larger, older seeds, but these conditions are typical of those used by farmers in many developing countries. Maximum weight gain in ponds was noted during optimum temperature while the lowest weight gain was observed during low temperature. The same results were obtained by Varadaraj [13]as some

Table 1. Fortnightly average body weight and weight gain in control and experimental ponds.

No. of Fortnights	Control		(50 mg17-α-methyltestosterone / kg feed)		(70 mg17-α-methyltestosterone / kg feed)		(90 mg17-α-methyltestosterone / kg feed)	
	Average weight (g)	Weight gain (g)	Average weight (g)	Weight gain (g)	Average weight (g)	Weight gain (g)	Average weight (g)	Weight gain (g)
1	20.2±1.50	-	22.1±2.10	-	22.3±1.10	-	24.0±2.20	-
2	29.9±1.30	9.7	33.0±2.20	11.8	34.2±2.11	11.9	36.1±2.15	12.1
3	38.2±2.10	8.3	44.5±2.30	10.6	45.0±1.14	10.8	47.7±2.11	11.6
4	45.7±2.15	7.5	54.5±2.20	10.0	55.3±1.18	10.3	58.4±2.13	10.7
5	52.4±2.25	6.7	69.2±2.30	9.7	65.3±2.10	9.9	68.1±2.24	9.7
6	58.5±2.15	6.1	71.4±2.20	7.2	73.6±2.30	8.3	76.6±2.30	8.5
7	63.6±3.25	5.1	77.7±2.10	6.3	81.0±1.50	7.4	84.1±2.20	7.5
8	67.9±3.05	4.3	81.9±2.15	4.2	87.3±1.80	6.3	90.9±2.15	6.8
9	71.5±2.50	3.6	85.4±3.10	3.5	92.6±2.40	5.3	97.7±2.25	6.8
10	74.0±2.30	3.0	88.5±3.20	3.1	96.7±2.30	4.1	103.8±1.90	8.1
11	76.7±2.20	2.7	90.9±3.15	2.4	100.2±2.10	3.5	107.4±1.95	3.6
12	79.0±2.10	2.3	93.1±3.20	2.2	103.6±2.10	3.4	112.3±2.20	4.9

factors like temperature affected the sex reversal of fry which increase the growth rate and survival of Tilapia. Smith and Phelps [16] also reported that fish activities including growth, development and metabolism highly depend upon temperature. There were prominent seasonal fluctuations but even then, water temperature remained favourable for fish growth. The water temperature remained lower than air temperature these results are in accordance with Mahboob et al [17].

5. CONCLUSIONS

The sex-reversed Tilapia exhibited maximum growth with the application of 90 mg/kg 17-alpha methyltestosterone. The Tilapia which produced through sex reversal has a capability to grow more as compared to un-treated Tilapia. It is suggested that future studies may consider using higher rates of the hormone dosage to evaluate the growth potential of sex reversed Tilapia.

6. REFERENCES

1. Khan, A.M., Z. Ali, S.Y. Shelly, Z. Ahmad & M.R. Mirza. Aliens; a catastrophe for native fresh water fish diversity. *Pakistan Journal of Animal and Plant Sciences* 21(2): 435-440 (2011).
2. Coward, K. & N.R. Bromage. Reproductive physiology of female tilapia broodstock. *Reviews in Fish Biology and Fisheries* 40(1): 1-25 (2000).
3. Phelps, R.P. Sex reversal: The directed control of gonadal development in tilapia. In: D.E. Meyer (Ed), *Proceedings for Tilapia Sessions from the 6th Central American Aquaculture Symposium* Tegucigalpa, Honduras, p. 35-60 (2001).
4. Guerrero, R.D. III & L.A. Guerrero. Feasibility of commercial production of sex-reversed Nile tilapia fingerlings in the Philippines. p. 183-86. In: R.S.V. Pullin, T. Bhukasawan, K. Tonguthai & J.C. Maclean (Eds.). *2nd International Symposium in Aquaculture*. ICLARM Conference Proc.15.

Department of Fisheries, Bangkok, Thailand and ICLARM, Manila, Philippines (1988).

5. Ahmad, M.H., A.M.E. Shalaby, Y.A.E. Khattab & M. Abdel-Tawwab. Effects of 17 α– methyltestosterone on growth performance and some physiological changes of Nile tilapia fingerlings (*Oreochromis niloticus* L.). *Egyptian Journal of Aquatic Fish Biology* 4(4): 295-311 (2002).
6. Dan, N.C. & D.C. Little. The culture performance of monosex and mixed-sex new-season and overwintered fry in three strains of Nile tilapia (*Oreochromis niloticus*) in northern Vietnam. *Aquaculture* 184: 221-231(2000).
7. Javed, M. & M.B. Sial. Fish pond fertilizatiom. III: Effect of layer manure fertilization on the growth performance of fish, viz. *Catla catla, Labeo rohita* and *Cirrhina mrigala*. *Pakistan Journal of Agriculture Science* 28(2): 115-120 (1991).
8. Steel, R.G.D., J.H. Torrie & D.A. Dinkkey. *Principles and Procedures of Statistics*, 2nd ed. McGraw Hill Book Co., Singapore (1996).
9. Boyd, C. E. *Water quality management for pond fish culture*. Elsevier Scientific Publishing Company, New York, USA, 318 pp. (1982).
10. Phelps, R.P. & T.J. Popma. Sex reversal of tilapia. In: B. A. Costa-Pierce & J. E. Rackocy (Eds.). *Tilapia Aquaculture in the Americas*, Vol. 2. The World Aquaculture Society, Baton Rouge, Louisiana, USA, p. 34-59 (2000).
11. Ridha, M.T., & K.P. Lone. Effect of oral administration of different levels of 17--A-methyltestosterone on the sex reversal, growth and food conversion efficiency of the Tilapia *Oreochromis spilurus* (Gunther) in brackish water. *Aquaculture and Fisheries Management* 21(4):391-397 (1990).
12. Hanson, R., R.O. Sinithevman. W.H. Shelton & R.A. Dunham. Growth comparison of monosex tilapia produced by separation of sexes hybridization and sex reversal, p. 570–579. In: L. Fishelson & Z. Yaron (compsi). *Proceedings of International symposium on Tilapia in Aquaculture,* 8-13 May 1983. Aviv University, Nazareth, Israel (1983).
13. Varadaraj, K., S.S. Kumari & T.J. Pandian. Comparison of conditions for hormonal sex reversal of *Tilapia mossambique*. *Progressive Fish-Culturist* 56(2): 81-90 (1994).
14. Carvalho, E.D. & F. Foresti. Sex reversal in Nile tilapia (*Oreochromis niloticus*) induced by treatment with 17-alpha-methyltestosterone: Sex ratio and histology of gonads. *Brazilian Journal of Biology* 56(2): 249-262 (1996).
15. David, C.L, C.B. Ram & A. P. Tuan. Advanced nursing of mixed-sex and mono-sex tilapia (*Oreochromis niloticus*) fry, and its impact on subsequent growth in fertilized ponds. *Aquaculture* 221: 265–276 (2003).
16. Smith, E.S. & R.P. Phelps. Impact of feed storage conditions on growth and efficiency of sex reversal of *Nile tilapia*. *North American Journal of Aquaculture* 63(2): 242-245 (2001).
17. Mahboob, S. & A.N. Sheri. Influence of fertilizers and artificial feed on the seasonal variation in physico-chemical factors in fish ponds. *Pakistan Journal of Zoology* 34(1): 51-56 (2002).

Proceedings of the Pakistan Academy of Sciences 51 (3): 215–233 (2014)

ISSN: 0377 - 2969 (print), 2306 - 1448 (online)

Pakistan Academy of Sciences

Review Article

Seaweed Antioxidants as Novel Ingredients for Better Health and Food Quality: Bangladesh Prospective

S. Bhattacharjee and G.M.R. Islam*

Department of Food Engineering and Tea Technology,
Shahjalal University of Science and Technology, Sylhet-3112, Bangladesh

Abstract: The known medicinal effects of seaweeds have been known for thousands of years. Also, a wide range of recent studies have described the high antioxidant capacity of a range of edible seaweeds. In this review article, the application of seaweed antioxidants in foods, food supplements, nutraceuticals and medicine is considered from the perspective of benefits to human health. The underlying physiology of algal antioxidant compounds is reviewed in the context of seaweed biology and utilization. It is found that direct consumption of seaweed products for their antioxidant composition provides a useful alternative to non-natural substances, while simultaneously providing worthwhile nutritional benefits. Economic utilization of seaweeds for their antioxidant properties is also inquired. Overall, the review confirmed that seaweeds antioxidants have huge potential in providing major health benefits through subsequent investigative studies relating to this in literature.

Keywords: Antioxidants, macroalgae, micronutrients, polyphenols, oxidative stress, ROS

1. INTRODUCTION

Seaweed is the common name for countless species of marine plants and algae that grow mostly on rocks and other plants in the ocean as well as in rivers, lakes, and other water bodies. It is chock-full of vitamins, minerals, and fiber, and can be tasty. In many countries, fresh seaweed are used as food by coastal communities, and considered as a traditional food item due to their nutritional value and characteristic [1, 2]. Seaweeds are a valuable source of proteins, polysaccharides and fiber; and they are also rich in antioxidants and micronutrients, such as vitamins and trace elements [3].

Among the most relevant components found in seaweeds are antioxidants which have attracted major interest due to their positive effects. Antioxidant vitamins and trace elements are usually obtained from the diet, since some organisms are unable to synthesize them. The beneficial effects of antioxidants are due to their capacity to scavenge and neutralize reactive oxygen species (ROS) [4] An excessive ROS production and/or low antioxidant defense can cause oxidative damage to biomolecules, such as proteins, lipids and DNA [8]. Antioxidants may reduce ROS production by scavenging free radicals through various mechanisms [4, 7].

Antioxidant has a potential to prevent cancer and cardiovascular diseases. The nature of these substances varies a lot whereas the most powerful antioxidants are polyphenols, phycobiliproteins, vitamin C, α-tocopherol and some carotenoids (xanthophylls). Furthermore, seaweed contains a high concentration of polysaccharides of various structure and functionality. The indigestible polysaccharides of macroalgae could be important sources of dietary fibres. These fibres can be insoluble such as cellulose, mannans and xylans or water soluble such as agar, alginic acid, laminarin, fucoidan and their derivatives which may potentially

Received, May 2014; Accepted, August 2014
*Corresponding author: G.M. Rabiul Islam, E-mail: rabiat14@yahoo.com, rabi-ttc@sust.edu

be exploited as prebiotics for applications in both human and animal health.

Seaweeds are a potentially good source of micronutrients and may be beneficial for human health, given their reportedly high vitamin and trace element content [1, 9]. Many published studies on seaweeds are focused on seaweed antioxidant vitamins and their medicinal effects. The objective of this study was to assess the potential of seaweeds as novel ingredients for better health and food quality in the Bangladesh perspective.

2. ECOLOGY AND USES OF SEAWEEDS

2.1. Ecology

Two specific environmental requirements dominate seaweed ecology. These are the presence of seawater (or at least brackish water) and the presence of light sufficient to drive photosynthesis. Another common requirement is a firm attachment point. As a result, seaweeds most commonly inhabit the littoral zone and within that zone more frequently on rocky shores than on sand or shingle. Seaweeds occupy a wide range of ecological niches. The highest elevation is only wetted by the tops of sea spray, the lowest is several meters deep. In some areas, littoral seaweeds can extend several miles out to sea. The limiting factor in such cases is sunlight availability. The deepest living seaweeds are some species of red algae.

A number of species such as Sargassum have adapted to a fully planktonic niche and are free-floating, depending on gas-filled sacs to maintain an acceptable depth. Others have adapted to live in tidal rock pools. In this habitat seaweeds must withstand rapidly changing temperature and salinity and even occasional drying.

2.2. Edible Seaweeds

Biologically, seaweeds are classified as macroalgae, with sub-classification as brown (Phaeophyta), red (Rhodophyta) or green algae (Chlorophyta). Some examples of these edible algae are outlined in Table 1. In 1994/95 over 2,000,000 tonnes (dry weight) of seaweed was harvested [10]. Much of this may be consumed as whole seaweed products, while a large proportion is also used in the production of over 85,000 tons of viscous polysaccharides for various food and industrial applications. Historically, seaweed is a readily available food source that has been consumed by coastal communities [11]. Seaweed is consumed habitually in many countries in South-East Asia [12]. However, it is not considered a habitual component of the Western diet [13].

Table 1. Eexamples of edible seaweeds.

Sub-classiicaton	Genus	Common Name
Brown Algae (Phaeophyta)	Alaria	Kelp/bladderlocks
	Himanthaia/ Bifurcaria	Sea spaghetti, fucales
	Limiliaria	Kelp/kombu/kumbu/ sea tangle
	Saccharina	Sugar wrack
	Uiularia	wakaae
	Ascophyllum	Egg wack
	Fucus	Bladder wack, rockweed
	Sargassum	Mojaban/Indian brown seaweed
	Hizikia	Hijiki
	Sargassum	Sea holly
	Diciyotales	
	Eisenia	Arame
Red algae (Rhodophyta)	Rhodymenia/ Palmaria	Dulse
	Porphyra	Noi/ haidai/ kim/ gim
	Chondrus	Irish moss/ carrigeen
	Mastocarpus/	Gigartiiia Stackhousc, Guir,
	Gracilaria	
	Asparagopsis	Limu Kohu
	Graleloupia	
Green algae (Chlorophyta)	Ulvaia/Enteroniorpha	Laver/sea lettuce/ sea grass/nori

In the West, seaweed isolates (e.g. alginate from brown algae and agar or carrageenan from red algae) are typically used industrially. Seaweed consumption has gained a measure of acceptance in some Westernized cultures such as Hawaii, California and Brazil, where there are large Japanese communities who have had a tangible influence on

the local dietary practices [14, 15]. Low consumer awareness regarding potential health benefits and a lack of previous experience of seaweed challenges its use in the daily diet.

Average Fresh Seaweed composition is 13% dry matter and 87% water. The dry matter is made of proteins, lipids, antioxidants, fiber and minerals (Fig.1).

2.3. Herbalism

Alginates are commonly used in wound dressings, and production of dental molds. In microbiology research, agar - a plant-based goo similar to gelatin and made from seaweed - is extensively used as culture medium. Carrageenans, alginates and agaroses (the latter are prepared from agar by purification), together with other lesser-known macro algal polysaccharides, also have several important biological activities or applications in biomedicine.

Seaweed is a source of iodine, necessary for thyroid function and to prevent goiter. However, an excess of iodine is suspected in the heightened cancer risk and even bigger risks in post-menopausal women. Thus Japanese people, who consume a lot of seaweeds (Fig. 2), have high risk of cancer.

Seaweeds may have curative properties for tuberculosis, arthritis, colds and influenza, worm infestations and even tumors [87]. Nori, a seaweed, is known as a remedy for radiation poisoning in Japan. Seaweed extract is also used in some diet pills [16]. Other seaweed pills exploit the same effect as gastric banding, expanding in the stomach to make the body feel fuller.

3. MAJOR ANTIOXIDANT COMPOUNDS IN SEAWEEDS

Most of the Seaweeds possess considerable antioxidant activity. However, presence of antioxidant doesn't completely ensure that a beneficial response will be the result of consumption of seaweeds by humans. The potential human health advantages depend upon both the respective intake of the plants, and the bioavailability of anticipated anti-oxidant activities [17]. Some examples of perceived health benefits related to specific antioxidant compounds from seaweeds are shown in Table 3. All energy-producing metabolic processes are intrinsically driven by an electron transport chain, maintenance of which is essential to the health and integrity of an organism. The hazards of a prolonged imbalance include formation of reactive species, unstable molecules or molecular fragments that, if not neutralized, can react with non-target molecules, causing a variety of (negative) cellular impacts [18]. These may include the initiation of increased cell proliferation, mitochondrial damage, excessive DNA strand breakage and deleterious chemical chain reactions leading to lipid peroxidation, enzyme inhibition and protein degradation [4, 8]. In healthy biological systems, reactive oxygen species (ROS) are continually produced. Alscher et al [18] highlighted

Table 2. The major groups of antioxidant compounds in seaweeds with specific examples and potential algal sources for utilization.

General Category	Example Compounds	Algal Source
Carotenoids	β-carotene	Chondrus crispus [19]
	Fucoxanthin	Mastocarpus stellatus Brown algae [20]
	Antheraxanthin,	Red algae [21]
	lutenin	
Phenolic compounds	Stypodiol, isoepitaondiol	Taonia atomaria [22]
	Terpenoids	Cystoseira sp. [23]
Phycobilin pigments	Phycoerythrin, phycocyanin	Red algae in general [24, 25]
Polyphenols	Catechin, gallate	Halimeda sp. [26]
	Flavonoids Phlorotannins	Palmaria palmate [27] Fucus vesiculosus
Sulphated polysaccharides	Fucoidan, alginic acid,	Turbinaria conoides [28]
	Fucoidan	Laminaria japonica [29]
	Sulphated-galactans	Some marine red algae [30]
	Galactans	Most red algae [31]
Vitamins	Ascorbate	Chondrus crispus [19]
		Mastocarpus stellatus Sargassum sp.
	Vitamin A	Kappaphycus alvarezii [32]

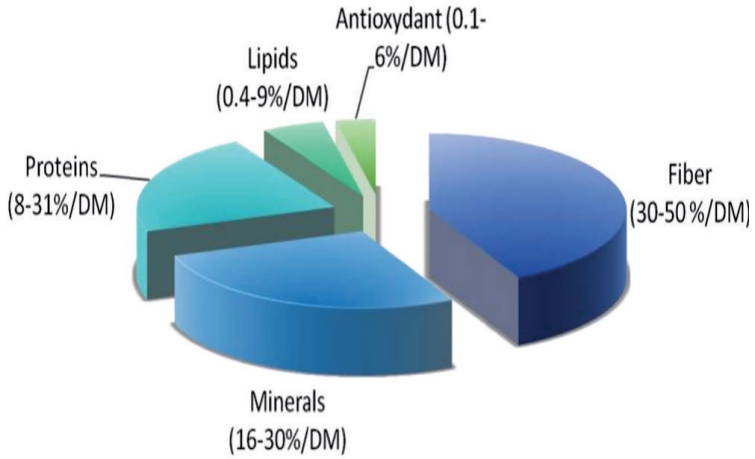


Fig. 1. Dry matter of antioxidant.

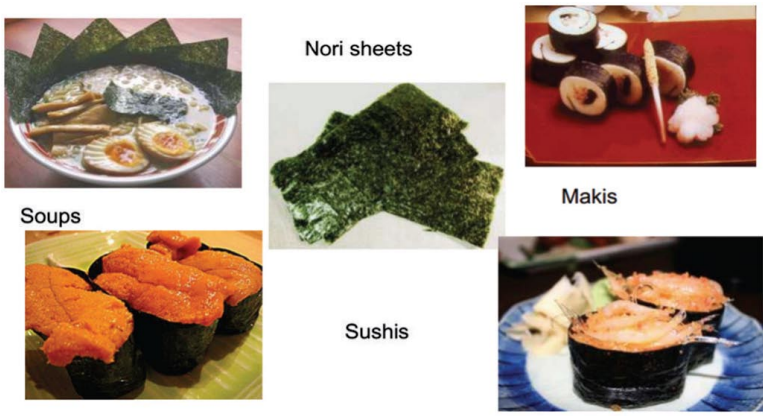


Fig. 2. Some popular seaweed based foods from Japan.

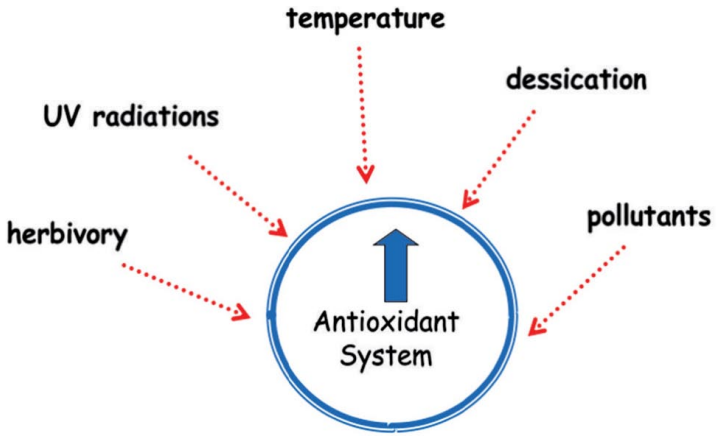


Fig. 3. Antioxidants : protective agents from various environmental stresses.

the role of reactive oxygen metabolism (ROM) in seaweeds, the stress factors that trigger it and details of the antioxidant response mechanisms.

The functional complexities associated with antioxidant defense mechanisms are diverse, and their relative importance against reactive species in vivo depends upon how, where and which reactive species (RS) is generated and what target of damage is measured. If commercialization of seaweeds for their antioxidant activity is to be considered, additional research is required to establish bioavailability of specific compounds [34] and to then guarantee production of standardized products containing them [35, 36]. This review will focus primarily on the potential health benefits and therapeutic properties purported to be associated with consuming seaweed and seaweed based products (Table 3).

Table 3. Salient examples of perceived health benefits of specific antioxidant compounds from seaweeds.

Antioxidant compound	Perceived health benefit	Reference
β-carotene, lutein	Antimutagenic	[37]
	Protective against breast cancer	[38]
Carrageenan	Anti-tumor	[39]
Fucoidan	Anti-HIV	[40]
	Anticancer	[41]
	Protection against neurodegenerative disorder	[29]
Fucoxanthin	Antiangiogenic, Protective effects against retinol deficiency	[42]
Galactan-sulfate	Anti-viral	[43]
Phlorotannins	Anti-inflammatory,	[27]
	Bactericide, Hypertension	[44]
	Vascular chemoprotection	[45]
Polyphenols	Antimicrobial	[46]

There is some concern about toxicological effects of seaweed. However, most edible seaweeds are marine algae whereas most freshwater algae are toxic. While marine algae are not toxic, some do contain acids that irritate the digestion canal, while some others can have a laxative and electrolyte-balancing effect [33].

4. SEAWEED AND HOMEOSTASIS

The stress-coping mechanisms of intertidal seaweeds are diverse and include antioxidant production, and free radical scavenging activities [19, 22, 47]. Two major stress coping mechanism include, maintaining the level of ROS and maintaining a metabolic oxidation / reduction balance. In [19], the author reported that the activity of three antioxidant enzymes, e.g. superoxide dismutase (SOD), ascorbate per-oxidase, and glutathione reductase in Chondrus crispus and Mastocarpus stellatus was greater in winter than in summer, suggesting that levels of reactive oxygen species (ROS) were also higher in winter as a result of cold stress. A gradual and continued accumulation of ROS in most macroalgae occurs as a result of environmental conditions such as dessication, freezing, low temperatures, high irradiance, ultraviolet radiation, heavy metals and salinity fluctuations [18, 19, 48, 49]. These stresses compromise photosynthesis, forming singlet oxygen that can cause damage to the photosynthetic apparatus [18]. To cope with such stresses, seaweeds deactivate the ROS by utilizing a high cellular content of antioxidant compounds, or by increasing the activity of antioxidant enzymes. This robust antioxidant potential of seaweeds helps minimize the hazardous effects of ultraviolet light or oxidation by ROS [50, 51].

Marine seaweed often experience exposure to high levels of both UVB and UVA radiation. While irradiance is required for the photosynthetic conversion of energy via light harvesting, electron transport, and ATP / NADPH synthesis, maintaining a metabolic oxidation / reduction balance is essential to the health and productivity of the system [52]. To quench the excess production of harmful radical species, seaweeds have evolved mechanisms such as photo-inhibition which leads to a slowly reversible reduction in photosynthetic rate from the maximum saturation level. This is brought about by either a reduction in the number of photosynthetic units, or by an increase in the maximum turnover time. The up-regulation of antioxidants and antioxidant enzymes, such as carotenoids and SODs and methods of cellular

repair by photo-reactivation and nucleotide excision are also strategies for maintaining homeostasis [47, 52]. In a comprehensive literature review [51] identified a number of compounds in marine algae to which antioxidant activity has been attributed. These included polyphenols, phycocyanins, various enzymes, carotenoids, catechins, and ascorbic acid (Table 1).

5. ANTIOXIDANTS IN HUMAN HEALTH

ROS, along with reactive nitrogen species (collectively labelled RS) have been identified as agents in various pathogenic diseases and deleterious clinical conditions related to human health. These include cancer, cardio-vascular disease, hemorrhagic shock, AIDS, atherosclerosis, hypertension, ischemia / re-perfusion, diabetes mellitus, hyperoxaluria, neuro-degenerative diseases such as Alzheimer's and Parkin-son's diseases, rheumatoid arthritis, ageing and even male-pattern baldness [6, 8, 46, 53-54]. The defense response to excess RS metabolism can involve preventative mechanisms, repair mechanisms and up-regulation of endogenous antioxidant defenses [8].

Melanoma and non-melanoma skin cancers are among the most prevalent cancers in the human population. They are often caused by large, or prolonged doses of UV radiation that overwhelm the natural protective antioxidant capacity of the skin [55, 56]. Using whole tissue ex-tracts in a naked mouse study, polyphenols derived from certain brown algae (e.g., Ecklonia spp.; see Appendix A) and applied either topically or administered through the diet provided highly protective effects against UVB induced skin carcinogenesis [57]. The dietary effects of non-seaweed derived commercial supplements of D-alpha-tocopherol and L-ascorbic acid on the sunburn reaction in humans is evaluated as a potential elicitor for skin cancer [58]. They determined that large doses of the two antioxidants acted synergistically to protect against sunburn damage. However, the effects of long-term administration of megadoses of these anti-oxidants requires more investigation.

In a study of female nurses and dietary intake of vitamins A, C, and E, folate and certain carotenoids, [59] could not conclusively demonstrate that these antioxidants protected against basal cell carcinoma un-der their experimental conditions. More recently, it is suggested that regular dietary anti-oxidant supplementation may even be associated with harmful effects, especially in women.

However, results of a two-year cohort study refuted this conclusion and that group observed no increased melanoma risk with supplementation of comparable doses of beta carotene and selenium [60]. Although these trials relate to non-seaweed sources of antioxidants, marine macroalgae possess complements of such active compounds in various amounts and ratios [51]. Ex-periments showed human and monkey cancer cell lines were inhibited by extracts of various seaweeds, especially by the brown algae Hydroclathrus clathratusand Padina arborescences [61]. The extracts, either in a crude state or after purification, demonstrated antioxidant activity and tumor suppression in a mouse model.

Cardiovascular disease (CVD) encompasses a broad range of primary and secondary conditions and its manifestation is a major cause of death −30%worldwide [5]. Risk factors for CVD include age, male gender, elevated low-density lipo-protein cholesterol levels, low high-density lipoprotein cholesterol levels, diabetes mellitus, smoking, chronic overeating and obesity. The adverse complications of obesity and unhealthy lifestyle factors are heightened by oxidative stress [62–64].

Extensive studies in pathophysiologic research clearly suggest that CVD represents a continuum of processes which include oxidative stress, endothelial dysfunction, inflammatory processes and vascular remodeling [64]. Foods rich in antioxidants have long been touted as aids in disease prevention. [65] assessed the association between the traditional Japanese dietary patterns and CVD. They concluded that a diet high in antioxidant foods, including seaweeds, de-creased the risk of CVD mortality. Kang et al [46] undertook an eight-

week human clinical trial to assess the effect of orally administered polyphenolic compounds. From brown algae on erectile dysfunction. Compounds from the five algae tested, Eisenia bicyclis, Ecklonia stolonifera, Ecklonia cava, Ecklonia kurome, and Hizikia fusi formis demonstrated positive effects against the risk factors associated with CVD. Deterioration of erectile function is a key in vivo indicator of cardiovascular health. Results from this trial showed significant improvement in erectile function and associated vascular health based on peripheral blood circulation.

Numerous studies into the synergistic effects of antioxidants and antioxidant enzymes and their interplay with RS suggest that the ideal protective mechanisms against clinical aspects of cellular damage should involve combinations, or whole suites of antioxidant compounds. Cellular homeostasis is thus more readily assured, and the possibility of profound imbalances brought about by high doses of single compounds can be effectively averted [55].

Considerable research demonstrates the human health benefits of naturally occurring antioxidant com-pounds. Claims of anti-viral, anti-inflammatory, anti-cancer, anti-mutagenic, anti-tumour, and hepatoprotective properties have been substantiated, albeit mostly from in vitro trials [27, 32, 57, 66-68] (Table 2).

6. FOOD VALUE AND HEALTH POTENTIAL OF SEAWEED

Seaweeds play an important role in this business which remains a growing, vibrant, and important production sector for healthy human food [90]. Intensive marketing programs and the popular health food press has recently raised the public profile of seaweed antioxidants considerably. However, clinical trials must be under-taken and publicized in order to educate and maintain consumer confidence. Aside from the direct health benefits, antioxidants from natural sources that combat lipid oxidation of foods, especially during processing and storage, are in high demand. The current use of synthetic antioxidants such as butylated hydroxyanisole, butylated hydroxy-toluene, and propylgallate is strictly regulated in many countries because they can in themselves pose potential health hazards, including carcinogenic effects [61, 69].

As part of a balanced diet, seaweeds can provide fibre, protein, minerals, vitamins and low fat carbohydrate content [27]. Seaweeds contain significant amounts of insoluble and soluble polysaccharides, and hence offer potential for fortification of food products with Dietary fibrs (DF) for technological and physiological purposes [89]. *The versatil*ity of seaweed as food allows consumption in fresh, dried, pickled or cooked forms and as a component in a wide assortment of other products. However, Seaweeds are eaten as whole foods by a relatively small percentage of the world population [27], in a relatively limited geography. Japanese form the largest consumer group eating on average, 1.6 kg dry weight per person, per year and they eat seaweed in great variety (fig.2). Scientists in Asian countries have demonstrated the health benefits derived from eating seaweeds [70], and the official Japanese Food Guide (see [71] for discussion) promotes seaweed as a nutritional foodstuff. Research is advancing into using marine macroalgae for production of novel foods, such as health beverages and processed meat products.

Several medicines and nutrients based on seaweeds are already available in market (Fig. 4, 5). The objective is to take advantage of their naturally occurring antioxidant compounds and other nutritive components [72]. This is a more holistic approach, based upon the observation that supplements of manufactured vitamins do not significantly decrease levels of oxidative damage in well-nourished individuals who already eat a balanced diet [5-6] . The extracted brown algal polyphenols from Eckloniasp decreases UVB-induced skin tumor development in mice regardless of whether the polyphenols were administered topically, or ingested as a dietary component, suggesting that the viability of these seaweed based antioxidants is unaffected by digestive processes [57]. A growing awareness of the functionality of

Fig. 4. Some Popular Seaweed based nutrients available in the market.

Fig. 5. Some medicines based on seaweed antioxidants.

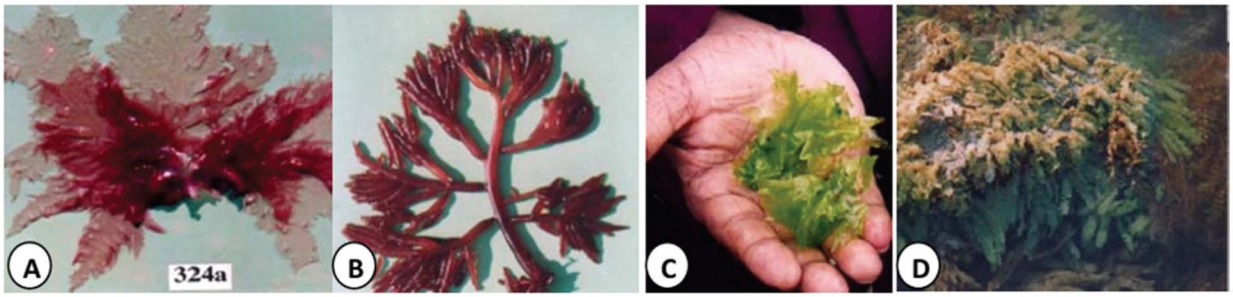


Fig. 6. Edibleseaweeds: (A) *Halymenia floridana*; (B) *Chrysymenia enteromorpha*; (C) *Ulva lactuca*, var. Rigidaand; (D) *Caulerpa racimosa*.

seaweeds beyond basic nutritive value will enhance the development of science and technology in this area of study.

7. ANTIOXIDANTS IN SEAWEEDS EXTRACTED USING COMPRESSED FLUIDS

In the search for feasible new sources of natural antioxidants that can be used in the food industry, seaweeds may be suggested as possible raw materials. These organisms are widely known and consumed in certain countries, and numerous health benefits have been associated with their use. Different compounds with antibacterial, antiviral and antifungal activity can be found in these types of organisms [73], along with compounds with antioxidant activity. Multifunctional antioxidant potential of several brown and red edible seaweeds has been evaluated in organic and aqueous soluble extracts and suitable sources of phytochemicals from seaweeds for further nutraceutical applications has been reported [88].

Spirulina is a blue-green microalga from Cyanobacterium gender. It is well known by its high content in proteins. Among them, phycobili proteins are known by its pharmaceutical and antioxidant properties. These proteins were characterized by using CE-ion trap-MS and CE-time of flight-MS. Later, the usefulness of CE-MS to monitor and optimize the pressurized liquid extraction of proteins from Spirulina platensis microalga was demonstrated [74]. The combined use of PLE and CE-MS allowed the attainment of extracts rich in phycobiliproteins in short extraction times (namely, yields of 20% can be obtained in less than 2 h under the optimum PLE process in an automatic way).

Moreover, Spirulina has been studied as an alternative source of functional ingredients, especially antioxidants. [75] studied the possibilities of using Spirulina supercritical extracts as both, antioxidants and antimicrobials. In this case, the best antioxidant extract was obtained in the first fraction when using intermediate pressures and temperatures (220-320 bar, 55 °C), with CO2 plus 10% ethanol as cosolvent, whereas higher pressures and temperatures (320 bar, 75 °C) were needed when pure CO2 was used. In order to provide a more in depth characterization of the antioxidant fraction, we used HPLCDAD-MS/MS with two different interfaces (ESI and APCI) that revealed the presence in the extracts of several carotenoids previously identified in Spirulina platensis microalga along with chlorophyll a and some degradation products [76]. Also, the structure of some phenolic compounds could be tentatively identified. Moreover the enrichment in vitamin E of extracts from microalga Spirulina platensis was studied [75]. The optimal conditions for the extraction of vitamin E from Spirulina platensis were achieved working at maximum temperatures, being the optimum value predicted, by using advanced statistical tools, of 29.4 mg/g extract which implies a tocopherol enrichment of more than 12 times the initial concentration of tocopherol in the raw material.

But also the subcritical fluid extraction of Spirulina has been studied in our research group. Different extraction temperatures (115°C and 170°C) and four different solvents (hexane, light petroleum, ethanol and water) were tested using extraction times ranging from 9 to 15 min. The antioxidant activity of the different extracts was determined by means of an in vitro assay using a free radical method. Moreover, a new and fast method was developed using micellar electrokinetic chromatography with diode array detection (MEKC-DAD) to provide a preliminary analysis on the composition of the extracts. It was observed that the optimum conditions that maximize yield and minimize EC50 depend on the polarity of the solvent used to perform the extractions. Extraction temperature had an enormous influence in both responses while the effect of extraction time was almost negligible. Ethanol was finally selected as the extracting solvent for its GRAS (Generally Recognized as Safe) status and because it provided higher yields with medium antioxidant activities [74]. Also, a new procedure was developed to separate and characterize antioxidant compounds

from Spirulina platensis microalgae based on the combination of pressurized liquid extraction(PLE) and different chromatographic procedures, such as TLC, at preparative scale, and HPLC-DAD [77]. TLC analysis of the best ethanolic extract obtained at 115°C for 15 min was carried out and the silica layer was stained with a DPPH solution to determine the antioxidant activity of different chromatographic bands. Next, these colored bands were collected for their subsequent analysis by HPLC-DAD, revealing that the compounds with the most important antioxidant activity present in Spirulina extracts were carotenoids, as well as phenolic compounds and degradation products of chlorophylls.

Another important microalgae well known by its antioxidant properties is *Dunaliella salina* [78]. This microalga has been studied in our research group considering both, supercritical fluid extraction and pressurized liquid extraction. In terms of PLE, the optimization of the extraction of antioxidants [74] and antimicrobials was carried out using experimental design with three different solvents (hexane, ethanol, and water) and two main factors (extraction temperature (40, 100, and 160°C) and extraction time (5, 17.5, and 30 min)). As response variables, the extraction yield (percent dry weight/initial weight) and the antioxidant activity of the extracts, determined using the trolox equivalent antioxidant capacity (TEAC) method, were considered. The parameters of the model were estimated by multiple linear regressions. Results showed that the extraction temperature was the factor having the strongest influence (positive) on the two response variables. The best yields were obtained with ethanol at the higher extraction temperature and time tested. Besides, although hexane extracts provided the best antioxidant activity, ethanol extracts were also very active. The chemical characterization of ethanol extracts was carried out using HPLC-DAD, and attempts were made to correlate their chemical composition with the antioxidant activity measured. Results pointed out that the extracts contained, besides all-trans-ß- carotene and isomers, several different minor carotenoids that seemed to make a strong contribution to the antioxidant activity of the extracts. By using supercritical CO2 different compositions of ß-carotene isomers were identified in the extracts by using HPLC-DAD. Also, antioxidant activity of the extracts was measured using a trolox equivalent antioxidant capacity (TEAC) assay. Higher yields were obtained at high pressures and low temperatures, that is, at higher CO_2 densities. Attempts were made to correlate the antioxidant activity of the extracts with their chemical composition by means of principal component analysis. A certain relationship was found between their antioxidant activity and the isomeric composition of ß-carotenes. As a result, an original equation was proposed to predict the antioxidant activity of extracts from *D. salina* in terms of the ratio 9-cis-ß-carotene/all-trans-ß-carotene, the concentration of a -carotene, and, especially, the concentration of 9-cis-ß-carotene.

8. SEAWEED AS A SOURCE OF OMEGA FATTY ACIDS

Apart from the abundance of minerals in seaweeds, it is their low, but very distinctive, fat content that attracts our attention. It can vary quite widely across the species, for example, being on the order of1-2% in dulse and konbu and up to 4-5% in wakame. Within a given species, it also depends on the time of year and the place where it grows. Most of the fats in seaweeds are made up of fatty acids with long hydrocarbon chains.

There are two relationships that are particularly interesting, namely, the relationship between the saturated and the mono- and polyunsaturated fats and that between omega-3 and omega-6 tatty acids.

A common trait of all seaweed species is that they contain about twice as much saturated as monounsaturated fat, but the combined total of unsaturated fat is greater than that of saturated fat. The crucial difference is the content of polyunsaturated fats, especially the super unsaturated fatty acids EPA (eicosapentaenoic acid) and AA (arachidonic acid), which are omega-3 and omega-6 fatty acids,

respectively. The polyunsaturated fats make up 30-70% of the total fat content, with omega-3 and omega-6 fatty acids accounting for most of it. It is an interesting point of comparison that no plants contain AA.

The most noteworthy aspect of the fat composition of seaweeds is the balance between the essential tatty acids omega-3 and omega-6. Essential fatty acids are those that our body cannot make and, therefore, has to ingest.

In the different species of seaweeds, the proportion of omega-3 to omega-6 typically falls between 0.3 and 1.8, with variations within a particular species, again dependent on where the seaweeds are grown and the time of year. From a nutritional standpoint, this is close to the ideal proportion for a human diet. Some nutritionists cautiously recommend that the figure should be about 0.2, but others think that it should be close to 1. , These recommendations should be contrasted with the proportion of 0.05 to 0.1, which is typical of the aver-age Western diet. As a consequence, this diet is far too rich in omega-6 fats and far too poor in omega-3 fat's.

Whereas seaweeds contain fair amounts off EPA (eicosapcntacnoicacid), the content of the other important omega-3 fatty acid found in the various seaweed species, DHA (docosahexaenoic acid), is often too small to be measured. This is in contrast to fish, which can have large quantities of both EPA and DHA. These two substances generally make up 30% and 20%, respectively, of the content of the fish oil sold as a dietary supplement. Unlike the macroalgae, the microalgac contain significant quantities of DHA in addition to FPA.

The omega-3 fatty acids found in fish and shellfish are not produced by these organisms themselves but obtained via the food chain from algae. Sterols are a particular type of fat, which seaweeds, like other higher organisms, utilize to strengthen their cell membranes. Two of these sterols, fucosterol and desmosterol, are elated to cholesterol. 'I he brown algae have an especially large sterol content, up to ten times as great as that in red algae. Normally, humans cannot make use of these seaweed sterols and less than 5% of the total is absorbed in the intestines. At the same time, however, they act to reduce the amount of cholesterol that is absorbed from other food. Studies have indicated that seaweed sterols help to decrease free and bound cholesterol levels and, in addition, to lower blood pressure.

Seaweeds contain up to 2% of dry weight of lipids and much of this lipid content is made up of polyunsaturated fatty acids (PUFA) [79, 80]. Table 4 illustrates that PUFAs account for almost half of this lipid content, with much of it occurring in the form of omega-3 and omega-6 lipids. The omega-3 to omega-6 ratio is closely matched, a factor that has been found to be important in a balanced diet. Both omega-3 and omega-6 fatty acids are essential, i.e., humans must consume them in the diet. Omega-3 and omega-6 compete for the same metabolic enzymes; thus, the omega-6:omega-3 ratio will significantly influence the ratio of the ensuing eicosanoids. This means omega-3 and omega-6 should be consumed in a balanced proportion, with the ideal ratio of omega-6:omega-3 ranging from

Table 4. Polyunsaturated fatty acid (PUFA) contents of seaweed*.

	Fatty Acid (% of Total fatty acid content)					
Seaweed	**Saturated**	**Monounsaturated**	**PUFAs**	**ω6 PUFAs**	**ω3 PUFAs**	**ω6:ω3 Ratio**
Himanthalia elongata	39.06	22.75	38.16	15.08	18.7	0.81
Laminaria ochroleuca	33.82	19.23	46.94	20.99	25.08	0.83
Undaria pinnatifida	20.39	10.5	69.11	22.1	44.7	0.49
Palmariaspp.	60.48	10.67	28.86	2.14	25.52	0.13
Porphyraspp.	64.95	18.91	16.1	7.97	7.2	1.21

***Source**: Sanchez-Machado et al [80].

3:1 to 5:1. Seaweeds contain many essential fatty acids, which may add to their efficacy as a dietary supplement or as part of a balanced diet [81]. Seaweeds are also normally tested after drying, but the effects of other types of food processing, such as canning, have been found to have a detrimental post-processing effect on fatty acid levels [80].

9. SEAWEED CULTURE: BANGLADESH PERSPECTIVE

The sophisticated technology and financial investment required to enter the emerging marine biotechnology market, results in dominance by wealthy countries. However, a focus on marine resources involving low-cost technology requirements, such as seaweed, provides an opportunity for developing countries like Bangladesh to access this emerging market. Seaweed aquaculture, growing at 7.5% per year, is becoming an important component of marine aquaculture, propelled by a diversification of the demand for seaweed products from traditional uses to bio-energy, cosmetics and biomedicine applications [91].

In Bangladesh, the natural abundance of commonly cultured tropical seaweeds of commercially important species is reported to be very low. Only small portion of the south-eastern part of the mainland covering only 30 km of the coast line in Ukiya and Teknaf and St. Martin Island have got rocky substratum and are suitable for naturally growing sea weeds.

Seaweeds are found mainly in the St. Martin Island and in mangrove forests. Seaweeds, having protein, amino acids, vitamins and minerals, are used as different purposes such as fodder, fertilizer, human food, industrial and pharmaceutical raw materials etc. Seaweeds reduce high blood pressure,

Table 5. Status of seaweed resources in Bangladesh.

S. No.	Species Scientific name	Abundance in Different Months					
		Type	Dec.	Jan.	Feb.	Mar.	Apr
1.	*Actinotrichia fragilis*	RSW	+	+	+		
2.	*Asperogapsis taxiformes*	RSW		+	++	++	+
3.	*Calliblepharis sp.*	RSW	++	+			+
4.	*Caulerpa sp.*	RSW	+	+	+	+	+
5.	*Ceramium sp.*	RSW	+		+	+	+
6.	*Chrysymenia sp.*	RSW		+	+	+	+
7.	*Cthonoplastis sp.*	RSW	+	++	++	++	++
8.	*Dictyota sp.*	BSW	+	+	+		+
9.	*Eucheuma sp.*	RSW		+			
10.	*Galauxara sp.*	RSW	+	+			+
11.	*Halymenia sp.*	RSW		+	+	+	+
12.	*Hydroclathara sp.*	BSW	++	+	+	+	+
13.	*Hypnea sp* (a)	RSW		+	+	++	++
14.	*Hypnea sp* (b)	RSW		++	+++	+++	+++
15.	*Hypnea sp* (c)	RSW		++	+++	+++	+++
16.	*Hypnea sp* (d)	RSW		+	++	++	++
17.	*Liagora sp*	RSW	+	+			+
18.	*Lobophara sp.*		+	+	+	+	+
19.	*Padina sp.*	BSW	+	+	+	+	+
20.	*Sargassum* (2 sp).	BSW		++	++	++	++
21.	*Scinnaia complanta*	RSW	+	+			+
22.	*Vanvorstrea coccinea*	RSW	+				

Source: Sarker [92]
RSW = Red Seaweeds; BSW = Brown Seaweeds
+ Normally available; ++ Moderately available; +++ Commercially available

cholesterol, and prevent strokes. They can also be used as remedy for rheumatism, diarrhea, and for controlling the growth of tumors. Study [82] and [83] reported 133 species of seaweed from the St.Martin's Island. [84] made 4 new records of seaweed from the coast of St. Martin's Island. And now more than 140 species of seaweed still found in the coastal area of Bangladesh. There are red and brown seaweed resources in Bangladesh. Among them the 5 most commercially important and available species of Bangladesh are *Hypnea sp., Cthonoplastis sp., Aspergapis taxiformes, Calliblepharis sp., Sargassum sp*.

Bangladesh have a variety of edible marine algae, such as *Halymenia floridana* (Fig. 6A), *Chrysimenia enteromorpha* (Fig. 6B), *Ulva lactuca var. rigida* (Fig. 6C) and *Caulerpa racimosa* (Fig. 6D). Researches should be carried out for growing these algae along the coastal belt starting from Teknaf to Sundarbans, using above mentioned algae as seed materials (inocula) from St. Martin Island adopting Chinese/Japanese technology. The St. Martin Island is being used as a cultivation site by most of the marine researchers of Bangladesh. To explore the full seaweed potential of Bangladesh, the south coastal belt starting from Teknaf to Sundarbans should be used, as the cultivation area of St. Martin Island is very small, compared to the population of Bangladesh.

Drifted seaweeds in the SMI are collected, dried and sold to neighboring country by local people. The people of St. Martin collect drifted seaweeds during Jan. to April, dry them in sun (Fig. 7A-B) and sell to Myanmar, at the rate of tk 5-8 per kg. The amount of harvest varies from 4-6 ton per day, being highest in March. There is a very good potentiality of cultivating edible seaweeds (which are present in the St. Martin) in the vast coastal areas of Bangladesh.

In recent years, sea weed cultivation has been an issue of much importance all over the world. Culture of sea weed is a maiden concept for the people of Bangladesh. The people of Bangladesh are very new about the systematic and cultivator techniques of seaweed. The red sea weed Hypnea sp. is widely distributed in the intertidal and sub-tidal water of St. Martin's island. A recent trial on Hypnea sp. cultivation in the St. Martin's island, funded by the SUFER (Support for University Fisheries Education and Research) project of Department for International Development (DFID), and Ministry of Science and Information and Communication Technology (MSICT), Government of Bangladesh, technically assisted by the Institute of Marine Sciences (IMU), University of Chittagong, has opened a new avenue in the aquaculture industry of the country.

Natural status quo and research endeavor show that Hypnea sp. (commonly known as maiden hair) is a fast growing species, found attached to objects like corals, stones, boulders, rope, bamboo, and even to other seaweed. One of the magnificent achievements of this research (which was carried out for the first time in Bangladesh) is that a number of locally adaptive seaweed culture technologies have been developed, which are easy to set-up and very much within the reach of the poor farmer. Another prominent feature of these seaweed culture technologies is that all the components used to prepare the culture systems are organic and eco-friendly.

However, this edible plant is used to prepare jellies. It has a mild flavor and delicate texture. Hypnea is high in carrageenan, which can thin the blood and lower cholesterol, helping to prevent strokes. It is also used against diarrhea. Compounds that stop the growth of tumors have been found in Hypnea. This sea plant also can be used as raw materials for pharmaceuticals and industrial products, animal feed or fertilizer.

A number of species of *Hypnea* sp. have so far been recorded from the cost of St. Martin's island. These are: *Hypnea boergesenii*, *H. coenomyce, H. musciformes, H. valentiae, H. cornuta*. Pertinently, *Hypnea sp*. culture in the coastal water of this country (especially along the inshore water of St. martin's Island) holds good promise for the farmers and private entrepreneurs. Already there exists a

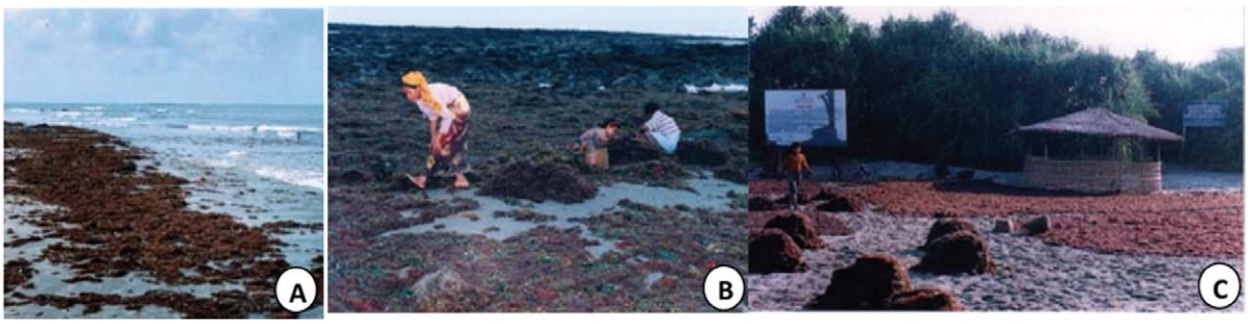


Fig. 7a-c. Seaweed Harvesting: A. Drifted seaweeds on the West coast of SMI. B. Members of a family collecting the seaweeds (mainly Hypnea and other red algae). C. Sun drying of seaweeds on the sandy beach.

Fig. 8a. Cultivated Hypnea sp. ; Fig. 12b. Growth observation *Hypnea sp.* on net in the Coastal water of St. Martin's Island.

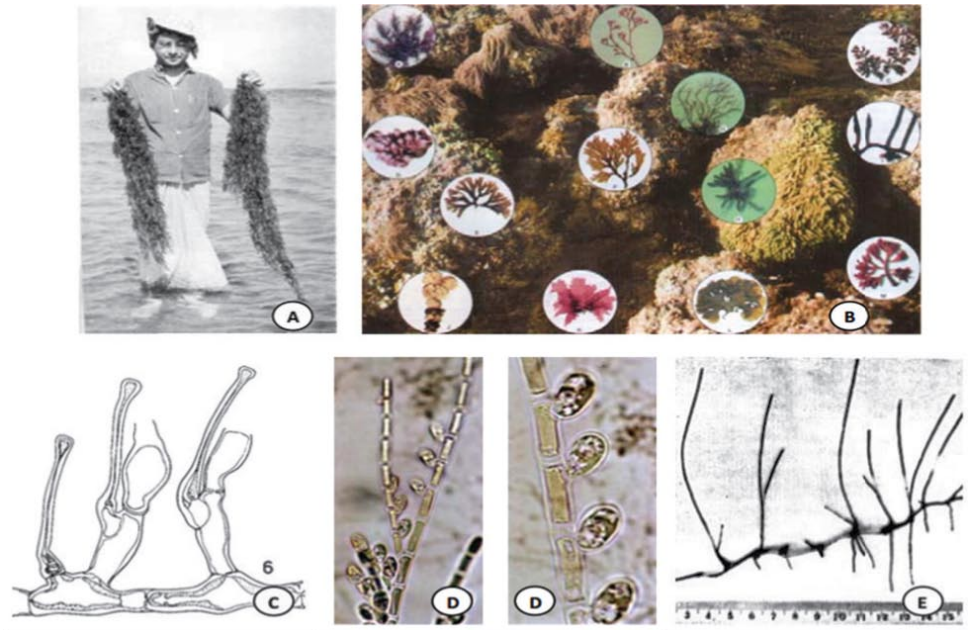


Fig. 9. Seaweeds and a "Sea grass" in the SMI. (A). A view of the seaweed flora in the west coast of SMI (B),1 March 1995 (right) Acrochaetium bengalicumIslametAziz (C). A. nurulislamiiA. Aziz et S. Islam (D) and Halodule uninervis Forskal (E).

seaweed marketing chain in the St. martin's Island. If initiatives are taken by the government and also by the privet agencies, a legal and stable marketing channel for seaweed can be established in the country.

In the marine environment it is the hard rocks, boulders, etc. where marine algae (seaweeds) grow. Thus it is the SMI, extended part of Teknaf and Inani beach where the seaweeds grow abundantly. All these collections and studies were made in the littoral zone (area exposed during low tide) of the St. Martin Island. But it appears that a lot more seaweeds occur in the sub-littoral zone of the area mentioned. The island is very rich in seaweeds and [85] have recorded two new species- Acrochaetium bengalicumIslam etAziz and A. nurulislamiiA. Aziz et S. Islam (Fig. 9C-D). Only two marine angiosperms, such as Halodule uninervis andHalophyla decipienswere recorded from the SMI (Fig. 9E).

Bangladesh can earn huge foreign exchange by exporting seaweed said Dr. Mohammad Zafor, prof of Institute of Marine Sciences and Fisheries of Chittagong University. In a research paper Dr. Zafar mentioned that, worldwide seaweed has been considered as one of the important economic resources [84]. It is used as industrial raw materials and edible item for human in many countries. It is reported that seaweed plays significant role in the economy of Japan. There are more than 140 species of seaweed found in the costal water of Bangladesh, especially in the St. Martin's Island. But these resources are a vital for maintaining ecological balance. Considering the nutritional and medicinal values of seaweed as well as it's possibility of being used as a human food, an attempt was taken to carry out research on seaweed cultivation in the inter tidal water of St. Martin's Island. If the initiative is taken to materialize this research breakthrough, coastal poor people can improve their economic and livelihood condition. Mass production of seaweed adds a new export item for Bangladesh.

10. CONCLUSIONS

The potential for commercialization of seaweed based, antioxidant compounds as food supplements or nutraceuticals ensures continued dedicated efforts to eventually develop functional, condition-specific, antioxidant products. Seaweeds are indeed suitable natural agents for producing and delivering these products based on the multi-functional aspects of secondary seaweed metabolites and the presence of a wide variety of associated non-toxic antioxidants [63, 86]. Such relatively non-toxic associations can enhance the synergistic effects of multiple antioxidants and provide buffering capacity if necessary for those compounds which may have been intentionally increased. Algae are efficient harvesters and proficient managers of electromagnetic energy and as highly nutritional food-stuffs, can be regularly consumed without fear of metabolic toxicities. We advocate the regular consumption of a variety of marine algae, primarily for their anticipated in vivo antioxidant capacities and associated synergistic effects. Rather than striving for targeted cause and effect mechanisms, which are developed in isolation and are generally fraught with the complexities of endogenous cellular activities, a diet rich in a diversity of seaweeds would provide healthy, whole food sustenance and competent antioxidant balance. Ideally, algae destined for human use would be derived from managed, sustainable sources, thus ensuring traceability and a high level of food safety and security. Core research avenues should include investigations into the bioavailability of seaweed based antioxidants [34]. Organisms, in general do not normally function in isolation at any metabolic level, and oxidation-reduction reactions and subsequent cellular exposures to RS are fundamental to all living things. It is the imbalance of RS that can compromise homeostasis, and it is a legion of relevant seaweed antioxidants that may mitigate, and even help reduce the impacts of cellular impairment.

Seaweed cultivation holds good promise in the coastal water of Bangladesh. Like other countries of the world, Bangladesh can boost its economic dimension through commencing seaweed cultivation. People should include it in their daily

food ration. Nutritional deficiency can well be met, if the people of Bangladesh will change their habit and consider seaweed as a comestible commodity. To materialize the seaweed research outcome, patronization from government sources and private entrepreneurs is a demand of the time.

11. REFERENCES

1. Chapman, V.J. Seaweeds and their uses, *Soil Science*. Chapman & Hall, London 74.6: 482 (1952).
2. Rupérez, P. Mineral content of edible marine seaweeds. *Food Chemistry* 79: 23-26 (2002).
3. Mamatha B.S., K.K. Namitha, A. Senthil, J. Smitha & G.A. Ravishankar. Studies on use of Enteromorpha in snack food. *Food Chemistry* 101: 1707-1713(2007).
4. Halliwell, B. & J.C. Gutteridge. Oxygen toxicity, oxygen radicals, transition metals and disease. *Biochemical Journal* 219.1: 1 (1984).
5. Halliwell, B. & J.C. Gutteridge. *Free Radicals in Biology and Medicine*, Vol. 135. Oxford University Press, Oxford, UK (1999).
6. Halliwell, B. Are polyphenols antioxidants or pro-oxidants? What do we learn from cell culture and in vivo studies? *Archives of Biochemistry and Biophysics* 476: 107-112 (2008).
7. Vertuani, S., A. Angusti & S. Manfredini. The antioxidants and pro-antioxidants network: an overview. *Current Pharmaceutical Design* 10.14: 1677-1694 (2004).
8. Valko, M., D. Leibfritz, J. Moncol, M.T. Cronin, M. Mazur & J. Telser. Free radicals and antioxidants in normal physiological functions and human disease. *The International Journal of Biochemistry & Cell Biology* 39.1: 44-84 (2004).
9. Cornish, M. L. & D.J. Garbary. Antioxidants from macroalgae: potential applications in human health and nutrition. *Algae* 25.4: 155-171 (2010).
10. Zemke-White, W.L. & M. Ohno. World seaweed utilisation: an end-of-century summary. *Journal of Applied Phycology* 11.4: 369-376 (1999).
11. Teas, J., J.R. Hebert, J.H. Fitton & P.V. Zimba. Algae–a poor man's HAART? *Medical Hypotheses* 62.4: 507-510 (2004).
12. Jiménez-Escrig, A., I. Jiménez-Jiménez, R. Pulido & F. Saura-Calixto. Antioxidant activity of fresh and pro-cessed edible seaweeds. *Journal of the Science of Food and Agriculture* 81: 530-534 (2001).
13. MacArtain, P., C.I. Gill, M. Brooks, R. Campbell & I.R. Rowland. Nutritional value of edible seaweeds. *Nutrition Reviews* 65.12: 535-543 (2007).
14. Cardoso, M. A., G.S. Hamada, J.M. de Souza, S. Tsugane & S. Tokudome. Dietary patterns in Japanese migrants to southeastern Brazil and their descendants. *Journal of Epidemiology/Japan Epidemiological Association* 7.4: 198 (1997).
15. Yamori, Y., A. Miura & K. Taira. Implications from and for food cultures for cardiovascular diseases: Japanese food, particularly Okinawan diets. *Asia Pacific Journal of Clinical Nutrition* 10.2: 144-145 (2001).
16. Maeda, H., M. Hosokawa, T. Sashima, K. Funayama & K. Miyashita. Fucoxanthin from edible seaweed, Undaria pinnatifida, shows antiobesity effect through UCP1 expression in white adipose tissues. *Biochemical and Biophysical Research Communications* 332.2: 392–397 (2005).
17. Manach, C., A. Scalbert, C. Morand, C. Rémésy & L. Jiménez. Polyphenols: food sources and bioavailability. *The American Journal of Clinical Nutrition* 79: 727-747 (2004).
18. Dring, M.J. Stress resistance and disease resistance in seaweeds: the role of reactive oxygen metabolism. *Advances in Botanical Research* 43: 175-207 (2005).
19. Lohrmann, N.L., B.A. Logan & A.S. Johnson. Seasonal acclimatization of antioxidants and photosynthesis in Chondrus crispus and Mastocarpus stellatus, two co-occurring red algae with differing stress tolerances. *The Biological Bulletin* 207.3: 225-232 (2004).
20. Sachindra, N. M., E. Sato, H. Maeda, M. Hosokawa, Y. Ni-wano, M. Kohno & K. Miyashita, Radical scavenging and singlet oxygen quenching activity of marine carotenoid fucoxanthin and its metabolites. *Journal of Agricultural and Food Chemistry* 55: 8516-8522 (2007).
21. Schubert, N., E. García-Mendoza & I. Pacheco-Ruiz. Carotenoid composition of marine red algae. *Journal of Phycology* 42: 1208-1216 (2006).
22. Nahas, R., D. Abatis, M.A. Anagnostopoulou, P. Kefalas, C. Vagias & V. Roussis. Radical-scavenging activity of Aegean Sea marine algae. *Food Chemistry* 102: 577-581 (2007).
23. Foti, M., M. Piattelli, V. Amico & G. Ruberto. Antioxidant activity of phenolic meroditerpenoids from marine algae. *Journal of Photochemistry and Photobiology B: Biology* 26.2: 159-164 (1994).
24. Romay, C. H., R. Gonzalez, N. Ledon, D. Remirez & V. Rimbau. C-phycocyanin: a biliprotein with antioxidant, anti-inflammatory and neuroprotective effects. *Current Protein and Peptide Science* 4.3: 207-216 (2003).
25. Sekar, S. & M. Chandramohan. Phycobiliproteins as a commodity: trends in applied research, patents and commercialization. *Journal of Applied Phycology* 20: 113-136 (2008).
26. Devi, K. P., N. Suganthy, P. Kesika & S.K. Pandian. Bio-protective properties of seaweeds: in vitro evaluation of antioxidant activity and antimicrobial activity against food borne bacteria in relation to polyphenolic content. *BMC Complementary and Alternative Medicine* 8.1: 38 (2008).
27. Yuan, Y.V., D.E. Bone & M.F. Carrington. Antioxidant activity of dulse (Palmaria palmata) extract evaluated in vitro. *Food Chemistry* 91.3: 485-494 (2005).
28. Chattopadhyay, N., T. Ghosh, S. Sinha, K. Chattopadhyay, P. Karmakar & B. Ray.

Polysaccharides from Turbinaria conoides: Structural features and antioxidant capacity. *Food Chemistry* 118.3: 823-829 (2010).
29. Luo, D., Q. Zhang, H. Wang, Y. Cui, Z. Sun, J. Yang, Y. Zheng, J. Jia, F. Yu, X. Wang & X. Wang. Fucoidan protects against dopaminergic neuron death in vivoand in vitro. *European Journal of Pharmacology* 617: 33-40 (2009).
30. Barahona, T., N. P. Chandía, M.V. Encinas, B. Matsuhiro & E.A. Zúñiga. Antioxidant capacity of sulfated polysaccharides from seaweeds. A kinetic approach. *Food Hydrocolloids* 25.3: 529-535 (2011).
31. Costa, L.S., G.P. Fidelis, S.L. Cordeiro, R.M. Oliveira, D.A. Sabry, R.B.G. Câmara & H.A.O. Rocha. Biological activities of sulfated polysaccharides from tropical seaweeds. *Biomedicine & Pharmacotherapy* 64.1: 21-28 (2010).
32. Kumar, K.S., K. Ganesan & P.V. Rao. Antioxidant potential of solvent extracts of Kappaphycus alvarezii (Doty) Doty–An edible seaweed. *Food Chemistry* 107.1: 289-295 (2008).
33. Wiseman, J. *SAS Survival Handbook: for any Climate in any Situation*. HarperCollins (2009).
34. Frankel, E.N. & J.W. Finley. How to standardize the multiplicity of methods to evaluate natural antioxidants. *Journal Agricultural and Food Chemistry* 56: 4901-4908 (2008).
35. Le Tutour, B., F. Benslimane, M.P. Gouleau, J.P. Gouygou, B. Saadan & F. Quemeneur. Antioxidant and pro-oxidant activities of the brown algae, Laminaria digitata, Himanthalia elongata, Fucus vesiculosus, Fucus serratus and Ascophyllum nodosum. *Journal of Applied Phycology* 10.2: 121-129 (1998).
36. Díaz-Rubio, M.E., J. Perez-Jimenez & F. Saura-Calixto. Dietary fiber and antioxidant capacity in Fucus vesiculosus products. *International Journal of Food Sciences and Nutrition* 60.S2: 23-34 (2008).
37. Okai, Y., K. Higashi-Okai, Y. Yano & S. Otani. Identification of antimutagenic substances in an extract of edible red alga, *Porphyra tenera* (Asakusa-nori). *Cancer Letters* 100: 235-240 (1996).
38. Maruyama, H., K. Watanabe & I. Yamamoto. Effect of dietary kelp on lipid peroxidation and glutathione per-oxidase activity in livers of rats given breast carcinogen DMBA. *Nutrition and Cancer* 15: 221-228 (1991).
39. Haijin, M., J. Xiaolu & G. Huashi. A k-carrageenan derived oligosaccharide prepared by enzymatic degra-dation containing anti-tumor activity. *Journal of Applied Phycology* 15: 297-303 (2003).
40. Béress, A., O. Wassermann, T. Bruhn, L. Béress, E.N. Kraiselburd, L.V. Gonzalez and P.I. Chavez. A new procedure for the isolation of anti-HIV compounds (polysaccharides and polyphenols) from the marine alga Fucus vesiculosus. *Journal of Natural Products* 56.4: 478-488 (1993).
41. You, S., C. Yang, H. Lee & B.Y. Lee. Molecular characteristics of partially hydrolyzed fucoidans from sporophyll of Undaria Pinnatifida and their in vitro anticancer activity. *Food Chemistry* 119.2: 554-559 (2010).
42. Sangeetha, R.K., N. Bhaskar & V. Baskaran. Comparative effects of β-carotene and fucoxanthin on retinol deficiency induced oxidative stress in rats. *Molecular and Cell Biochemistry* 331: 59-67 (2009).
43. Talarico, L.B., R.G. Zibetti, P. Faria, L.A. Scolaro, M.E. Duarte, M.D. Noseda & E.B. Damonte. Anti-herpes simplex virus activity of sulfated galactans from the red seaweeds Gymnogongrus griffithsiae and Cryptonemia crenulata. *International Journal of Biological Macromolecules* 34.1: 63-71 (2004).
44. Nagayama, K., Y. Iwamura, T. Shibata, I. Hirayama & T. Nakamura. Bactericidal activity of phlorotannins from the brown alga Ecklonia kurome. *Journal of Antimicrobial Chemotherapy* 50: 889-893 (2002).
45. Cha, S.H., K.W. Lee & Y.J. Jeon. Screening of extracts from red algae in Jeju for potential marine angio-tensin-I converting enzyme (ACE) inhibitory activity. *Algae* 21: 343-348 (2006).
46. Kang, K., Y. Park, H.J. Hwang, S.H. Kim, J.G. Lee & H.C. Shin. Antioxidative properties of brown algae poly-phenolics and their perspectives as chemopreventative agents against vascular risk factors. *Archives of Pharmacal Research* 26: 286-293 (2003).
47. Martínez, R. Effects of ultraviolet radiation on protein content, respiratory electron transport system (ETS) activity and superoxide dismutase (SOD) activity of Ant-arctic plankton. *Polar Biology* 30: 1159-1172 (2007).
48. Harker, M., C. Berkaloff, Y. Lemoine, G. Britton, A. Young, J.C. Duval & B. Rousseau. Effects of high light and desiccation on the operation of the xanthophyll cycle in two marine brown algae. *European Journal of Phycology* 34.1: 35-42 (1999).
49. Connan, S., E. Deslandes & E.A. Gall. Influence of day–night and tidal cycles on phenol content and antioxidant capacity in three temperate intertidal brown seaweeds. *Journal of Experimental Marine Biology and Ecology* 349.2: 359-369 (2007).
50. Garbary, D. The margin of the sea. In: *Algae and Cyanobacteria in Extreme Environments*, p. 173-191. Springer, Netherlands (2007).
51. Yoshiki, M., T. Tsuge, Y. Tsuruta, T. Yoshimura, K. Koganemaru, T. Sumi, T. Matsui & K. Matsumoto. Production of new antioxidant compound from mycosporine-like amino acid, Porphyra-334 by heat treatment. *Food Chemistry* 113: 1127-1132 (2009).
52. Sinha, R.P., M. Klisch, A. Gröniger & D.P. Häder. Ultraviolet-absorbing/screening substances in cyanobacteria, phytoplankton and macroalgae. *Journal of Photochemistry and Photobiology B: Biology* 47.2: 83-94 (1998).
53. Park, D.S., K.H. Lee, H.C. Kim, M.J. Ahn, C.J. Moon, M.S. Ko, K.K. Lee, G.M. Go & T. Shin. Effects of Callophyllis japonica powder on carbon tetrachloride-induced liver injury in rats. *Oriental Pharmacy and Experimental Medicine* 5: 231-235

(2005).
54. Veena, C.K., A. Josephine, S.P. Preetha & P. Varalakshmi. Beneficial role of sulfated polysaccharides from edible seaweed Fucus vesicuolosusin experimental hy-peroxaluria. *Food Chemistry* 100: 1552-1559 (2007).
55. Steenvoorden, D. P. T. & G.M.J.B. van Henegouwen. The use of endogenous antioxidants to improve photo-protection. *Journal of Photochemistry and Photobiology B: Biology* 41: 1-10 (1997).
56. Sander, C.S., H. Chang, F. Hamm, P. Elsner & J.J. Thiele. Role of oxidative stress and the antioxidant net-work in cutaneous carcinogenesis. *International Journal of Dermatology* 43: 326-335 (2004).
57. Hwang, J.S., S. Souissi, L.C. Tseng, L. Seuront, F.G. Schmitt, L.S. Fang. A 5-year study of the influence of the northeast and southwest monsoons on copepod assemblages in the boundary coastal waters between the East China Sea and the Taiwan Strait. *Journal of Plankton Research* 28: 943-958 (2006).
58. Fuchs, J. & H. Kern. Modulation of UV-light-induced skin inflammation by D-alpha-tocopherol and L-ascor-bic acid: a clinical study using solar simulated radiation. *Free Radical Biology and Medicine* 25: 1006-1012 (1998).
59. Fung, T.T., D.J. Hunter, D. Spiegelman, G.A. Colditz, F.E. Speiz-er & W.C. Willett. Vitamins and carotenoids intake and the risk of basal cell carcinoma of the skin in women (United States). Cancer Causes Control, 13: 221-230 (2002).
60. Asgari, M. M., S.S. Maruti, L.H. Kushi & E. White. Antioxidant supplementation and risk of incident melanomas: results of a large prospective cohort study. *Archives of Dermatology* 145.8: 879 (2009).
61. Wang, T., R. Jónsdóttir & G. Ólafsdóttir. Total phenolic compounds, radical scavenging and metal chelation of extracts from Icelandic seaweeds. *Food Chemistry* 116: 240-248 (2009).
62. Oben, J., E. Enonchong, D. Kuate, D. Mbanya, T.C. Thomas, D.J. Hildreth, T.D. Ingolia & M.S. Tempesta. The effects of ProAlgaZyme novel algae infusion on metabolic syndrome and markers of cardiovascular health. *Lipids, Health and Disease* 6-20 (2007).
63. Bocanegra, A., S. Bastida, J. Benedí, S. Ródenas & F.J. SánchezMuniz. Characteristics and nutritional and cardiovascular-health properties of seaweeds. *Journal of Medicinal Food* 12: 236-258 (2009).
64. Riccioni, G. Carotenoids and cardiovascular disease. *Current Atherosclerosis Reports* 11: 434-439 (2009).
65. Shimazu, T., S. Kuriyama, A. Hozawa, K. Ohmori, Y. Sato, N. Nakaya & I. Tsuji. Dietary patterns and cardiovascular disease mortality in Japan: a prospective cohort study. *International Journal of Epidemiology* 36.3: 600-609 (2007).
66. Yan, X., Y. Chuda, M. Suzuki. & T. Nagata. Fucoxanthin as the major antioxidant in Hijikia fusiformis, a common edible seaweed. *Bioscience and Biotechnology Biochemistry* 63: 605-607 (1999).
67. Yan, X., T. Nagata & X. Fan.. Antioxidative activities in some common seaweeds. *Plant Foods for Human Nutrition* 52.3: 253-262 (1998).
68. Lim, Y. Y., & J. Murtijaya. Antioxidant properties of Phyllanthus amarus extracts as affected by different drying methods. *LWT-Food Science and Technology* 40.9: 1664-1669 (2007).
69. Matanjun, P., S. Mohamed, N.M. Mustapha, K. Muhammad & C.H. Ming. Antioxidant activities and phe-nolics content of eight species of seaweeds from north Borneo. *Journal of Applied Phycology* 20: 367-373 (2008).
70. Nisizawa, K. Seaweeds Kaiso: bountiful harvest from the seas. *World Seaweed Resources-An Authoritive Reference System 90*.75000: 80 (2006).
71. Rhatigan, P. Prannie *Rhatigan's Irish Seaweed Kitchen: The Comprehensive Guide to Healthy Everyday Cooking with Seaweeds*. Booklink *(*2009*)*.
72. López-López, I., S. Bastida, C. Ruiz-Capillas, L. Bravo, M.T. Larrea, F. Sánchez-Muniz & F. Jiménez-Colmenero. Composition and antioxidant capacity of low-salt meat emulsion model systems containing edible seaweeds. *Meat Science* 83.3: 492-498 (2009).
73. Cohen, Z. *Chemicals from Microalgae*. CRC Press (1999).
74. Herrero, M., P.J. Martín-Álvarez, F.J. Senorans, A. Cifuentes & E. Ibáñez. Optimization of accelerated solvent extraction of antioxidants from Spirulina platensis microalga. *Food Chemistry* 93.3: 417-423 (2005).
75. Mendiola, J. A., D. García-Martínez, F.J. Rupérez, P.J. Martín-Álvarez, G. Reglero, A. Cifuentes & F.J. Señoráns. Enrichment of vitamin E from Spirulina platensis microalga by SFE. *The Journal of Supercritical Fluids* 43.3: 484-489 (2008).
76. Mendiola, J. A., F.R. Marín, S. Hernandez, B.O. Arredondo, F.J. Señoráns, E. Ibañez & G. Reglero. Characterization via liquid chromatography coupled to diode array detector and tandem mass spectrometry of supercritical fluid antioxidant extracts of Spirulina platensis microalga. *Journal of Separation Science* 28(9-10): 1031-1038 (2005).
77. Jaime, L., J.A. Mendiola, M. Herrero, C. Soler-Rivas, S. Santoyo, F.J. Señorans & E. Ibáñez. Separation and characterization of antioxidants from Spirulina platensis microalga combining pressurized liquid extraction, TLC, and HPLC-DAD. *Journal of Separation Science* 28.16: 2111-2119 (2005).
78. Chidambara Murthy K.N., A. Vanitha, J. Rajesha, M. Swamy Mahadeva, P.R. Sowmya, G.A. Ravishankar. In vivo antioxidant activity of carotenoids from Dunaliella salina- a green microalga. *Life Sciences* 76:1381-1390 (2005).
79. Marsham, S., G.W. Scott & M.L. Tobin. Comparison of nutritive chemistry of a range of temperate seaweeds. *Food Chemistry* 100.4: 1331-1336 (2007).
80. Sanchez-Machado, D., J. Lopez-Hernandez, P. Paseiro-Losada & J. Lopez-Cervantes. Fatty acids, total lipid, protein and ash contents of processed

edible seaweeds. *Food Chemistry* 85: 439–444 (2004).

81. Dembitsky, V.M., H. Řezanková, T. Řezanka & L.O. Hanuš. Variability of the fatty acids of the marine green algae belonging to the genus Codium. *Biochemical Systematics and Ecology* 31.10: 1125-1145 (2003).
82. Islam, A.K.M. Contribution to the study of the marine algae of Bangladesh. *Bibliotheca Phycologica* 19: 1-253 (1976).
83. Islam, A.K.M.N. and A. Aziz. Addition to the list of marine algae of St. Martins Island, Bangladesh II. Brown, red and blue green algae. *Nova Hedwigia* (34) (1982).
84. Zafar, M. *Seaweed Culture (Hypnea sp.) in Bangladesh* (unpublished, 2007).
85. Aziz, A. Coastal and marine plant resources of Bangladesh: management and exploitation. *Keynote Paper, Annual Botany Conference*. University of Chittagong, Chittagong, Bangladesh (2010).
86. Smit, A. J. Medicinal and pharmaceutical uses of sea-weed natural products: a review. *Journal of Applied Phycology* 16: 245-262 (2004).
87. Ashok. K., K. Sivakumari & P. Jayaprakash. Seaweeds. *International Journal of Applied Biochemistry* 1.1: 30-36 (2013).
88. Jiménez-Escrig, A., E. Gómez-Ordóñez & P. Rupérez. Brown and red seaweeds as potential sources of antioxidant nutraceuticals. *Journal of Applied Phycology* 24.5: 1123-1132 (2012).
89. Borderías, A. J., M.P. Pérez-Mateos & I. S. Alonso. Fibre-rich and wholegrain foods: improving quality. p. 348-368. *Woodhead Publishing Series in Food Science, Technology and Nutrition* No. 237 (2013).
90. Wiencke, C. & K. Bischof. *Seaweed Biology: Novel Insights into Ecophysiology, Ecology and Utilization* (Vol. 219). Springer (2012).
91. Mazarrasa, I., Y.S. Olsen, E. Mayol, N. Marbà, & C.M. Duarte. Rapid growth of seaweed biotechnology provides opportunities for developing nations. *Nature Biotechnology* 31.7: 591-592 (2013).
92. Sarker, M.N. Status of the red seaweeds in Bangladesh. In: *Regional Workshop on the Taxonomy, Ecology and Processing of Commercially Important Red Seaweeds*, 21–28 April, Bangkok, Thailand. Organized by FAO, NACA and France Government (1992).

Proceedings of the Pakistan Academy of Sciences 51 (3): 235–240 (2014)

ISSN: 0377 - 2969 (print), 2306 - 1448 (online)

Pakistan Academy of Sciences

Research Article

Pharmacotherapy versus Psychotherapy in the Management of Male Bipolar Disorder at Pakistan Institute of Mental Health, Lahore, Pakistan

Hira Ijaz[1*], Junaid Qureshi[1], Muhammad Zaman[2], Imran Waheed[3,4], Imran Khan[5] and Mohamed Abdel-Daim[6]

[1]College of Pharmacy, Government College University, Faisalabad, Pakistan
[2] Faculty of Pharmacy, The University of Lahore, Lahore, Pakistan
[3]Lahore Pharmacy College, Lahore, Pakistan
[4] Research Institute of Pharmaceutical Sciences, Department of Pharmacognosy, University of Karachi, Karachi, Pakistan
[5]Department of Cardiac Surgery, Punjab Institute of Cardiology, Lahore, Pakistan
[6]Pharmacology Department, Faculty of Veterinary Medicine, Suez Canal University, Ismailia, Egypt

Abstract: This exploratory study examined effectiveness of pharmacotherapy and psychotherapy treatment models to examine the factors contributing non-compliance of male bipolar disorder (BD) patients in the Punjab Institute of Mental Health (PIMH), Lahore, Pakistan and their impact on patients' quality of life. The data were collected from 100 patients. The 40-days data were collected from 12th January, 2014 to 22nd February 2014. The study revealed that psychotherapy is used just on and off (5%), pharmacotherapy alone in 35% cases and combination of both the therapies are used most commonly(60%). The results support the fact that the pharmacotherapy and psychotherapy both are helpful in treatment of bipolar disorder patient where as pharmacist plays an important and central role in treatment of BD by using pharmacotherapy interventions.

Keywords: Pharmacotherapy, psychotherapy, bipolardisorder

1. INTRODUCTION

Over the past 25 years, the role of pharmacist in the public and nonprofit sector has become limited to writing prescriptions in state and county institutions, community mental health centers, and other organized service settings, with most nonprofit health maintenance organizations following suit [1-2]. In the past decade, psychiatrists in the expanding managed private for-profit sector have been increasingly used as pharmacotherapists, while psychotherapy services have been provided by nonphysician mental health specialists [3-5].

According to WHO figures, in developing countries like Pakistan, one percent of the population suffers from severe and 10% from mild mental disorders [5-6]. According to the Global Burden of Disease (GBD) the mental illnesses constitute 10.5% of GBD, which may rise up to 15% in the year 2020 [6-7]. Among the top ten major causes of disability, five are mental illnesses, contributing 29% of the total disabilities while behavioral problems contribute an additional 34% to the GBD. These figures do not include cases of mental retardation and drug addicts [5, 8-9]. It has been estimated that there are 5.1 million chronic drug abusers in Pakistan and according to NSDA (National Survey on Drug Abuse), 51% of the addicts are dependent on opiates particularly on Heroin [10-12]. The

Received, April 2014; Accepted, August 2014
*Corresponding Author: Hira Ijaz, E-mail: pharmacisthira@gmail.com

importance and gravity of the problems related to mental illnesses can well be estimated from the facts based on scientific studies by WHO that two-fifths (40%) of total disabilities at global scale are due to mental illnesses effecting human functionality which may disturb socio-economic scenario of any country [13-16] .

In recent years, incidences of psychological disorders are increasing day by day [17]. The aim of present study was to evaluate patient characteristics, including history, social, cognitive, and work function, for prediction of the outcome of major depressive disorder. They provide indirect evidence of treatment specificity by identifying characteristics responsive to different modalities, which may be of value in the selection of patients for treatment. Main objective of this study was to evaluate the effectiveness of psychological intervention and pharmacotherapy interventions or combination of both.

2. METHODOLOGY

The study included treatment strategies given to male patients with bipolar disorder, data was collect from inpatient record as well from currently treated patients and by discussing it with experts (consultants, psychologists and hospital pharmacist).

Sample Size

Data from 100 patients who had bipolar disorder from medical record of male BD patients. Treatment plan given to those patients was studied in Pakistan Institute of Mental Health Lahore, Lahore, Pakistan.

Measures

40 days data was collected from 12-1-2014 to 22-2-2014.

Procedure

Study was carried out under these steps. Patient profile and their past medical history were studied. Laboratory findings were observed according to the treatment plan and physiological condition of patient. Different treatment choices were observed and evaluated rational use of pharmacotherapy. Quality of care was observed with respect to patient whether patient was properly treated by medical staff and doctors. Pharmaceutical care was observed and analyzed with respect to patient treatment plan and medication dispensing. Treatment plan was observed and analyzed with respect to WHO guidelines.

Patient Elligibility Criteria

Inclusion Criteria

- In patients with bipolar disorder
- Bipolar affective disorder

Exclusion Criteria

- Patient who were not BIPOLAR were not selected.
- Patients with age <18 years were not selected.
- Patients with any other mental disorder
- Patients who take antidepressants prophylactically

Method of Study

i. ***Quantitative Methods***

Sample survey

ii. **Qualitative Methods**

A structured questionnaire.

3. RESULTS

The criteria for data collection form was extracted from the guidelines of NIMH.

Out of 100 patients, 28 were between 20-30 years of age, 48 were between 30-40 years, 18 patients were between 40-50 years, and 6 were above 50 years having past medical history. 20% patients were un-educated, 42% had primary education, 6% had middle level, 24% had completed matriculation and 8% had passed intermediate level education. Out of 100 patients, 21 patients had family history and 79 had no family history. 38 patients had continuous course of illness and 62 patients had intermittent course of illness. 23% patient had euthymic, 68% had irritable and 9% had hypertymic temperament. 100% patients were educated by pharmacist on how to cope with

diseases. Drug interaction showed 34% patients had major drug interactions, 45 patients had minor, 12 had rare and 9 had no interaction. Most of the patients used antipsychotics. Most commonly used medications are resperidone and lorazepam. Psychomotor activity and patient compliance were also observed, as shown in Table 1. Provisional disgnosis showed 27% patients has BA, 48% had BP-1 and 25% had manic depressive illness. Most of patients received combinations of both psychotherapy and pharmacotherapy interventions. Between 75% patients had hallucination and 25% had no hallucinations Most of the patients were co-operative.13% patients had no previous admitions in PIMS.

4. DISCUSSION

According to American Psychiatric Association, pharmacotherapy is the first-line offense against episodes of bipolar disorder [18-19]. In recent study attempt was made to examine the impact of psychotherapy and pharmacotherapy for treatment of bipolar disorder patients. We specify the inclusion and exclusion criteria which help in selection of patients. We found the consistent decrease in number severity of condition when pharmacotherapy and combination of both pharmacotherapy and psychotherapy was used. Similar findings were also observed by Paola 2002 while studying the effectiveness pharmacotherapy and psychotherapy in treatment of bipolar disorder [20].

Although this study examined the effects of pharmacotherapy versus psychotherapy treatment on health care utilization, ultimately the more important question is the effectiveness of different models of treatment for different groups of patients [21]. Research should address the questions of which patients benefit most from which treatment models, including the broadly conceptualized models of pharmacotherapy versus psychotherapy. A better understanding of the qualitative differences between the two models of treatment and how the two models are structured might help explain the observed differences [20-22]. Some evidence exists for the suggestion that subtypes of patients with bipolar disorder might benefit selectively from different treatment strategies, and given the controlled nature of managed care settings, different factors related to health systems must be taken into account [19].

Main objective of our project was to study about different therapies used in bipolar disorder, relapse of BD, social support, attitude of family and relatives towards BD patients and most important patient compliance. Throughout our study we observed the attitude of doctors, staff, psychiatrics, pharmacists towards patients and their contribution in patient prognosis improving his quality of life, as far as PIMH is concerned. We observed keenly consultants were all in all authority in treating patients, psychologists provide therapies only when needed but were not playing a major role. Staff members are much less than demand as patients are many in number.

PIMH is the largest institute in Asia with 1400 beds and only 3 pharmacists were appointed, which is insufficient so atleast WHO standards of 1 pharmacist for 200 beds should be fulfilled for proper source of guidance. Role of pharmacist in management of BD patient, which was unfortunately nil. Pharmacists were only providing distributive services, no pharmaceutical care nor pharmacovigilence was provided . Clinical pharmacist is entirely absent in PIMH but as we observed during our study that ratio of drug interaction was very high which need pharmaceutical intervention and TDM is required. One of the reasons of this lacking is due to the shortage of pharmacists.

The situation in Pakistan regarding improvement in mental health services is not at the pace to reach a satisfactory level. This important field of health is not popular, as it should be if we compare it to some other medical and surgical disciplines such as cardiology and ophthalmology, etc.

5. CONCLUSIONS

Because medication and psychotherapy play central

Table 1. Characteristics of the patients with BD who received pharmacotherapy and psychotherapy in PIMH, Lahore, Pakistan.

Characteristic	N = 100	Percent (%)
Past Psycatric History	28 (20-30)	28
	48 (30-40)	48
	18 (40-50)	18
	6 (Above 50)	6
Education	20 (Nil)	20
	42 (Primary)	42
	6 (Middle)	6
	24 (Matriculation)	24
	8 (Intermediate)	8
Family History	21(Yes)	21
	79 (No)	79
Past Medical History	24 (20-30)	24
	52 (30-40)	52
	14 (40-50)	14
	10 (Above 50)	10
Duration of Present Complaint	24 (20-30)	24
	32 (30-40)	32
	34 (40-50)	34
	10 (Above 50)	10
Course of Illness	62 (Continuous)	62
	38 (Intermittent)	38
Temperament of Patient	23 (Euthymic)	23
	68 (Irritable)	68
	9 (Hyperthymic)	9
Patients Educated by Pharmacist (that how to cope with his disorder?)	100	100
Type of Drug Interaction	34 (Major)	34
	45 (Minor)	45
	12 (Rare)	12
	9 (None)	9
Class of Antipsychotics Drugs being Used	44 (Antipsychotics)	44
	34 (Antidepressants)	34
	19 (Anxiolytics)	19
	3 (Antiepileptic & mood stabilizers)	3
Drugs Commonly Used	24 (Riserperidone)	24
	20 (Haloperidol)	20
	21 (Fluphenazine)	21
	23 (Lorazepam)	23
	12 (Chloropromazine)	12

contd...

Table 1 *(contd...)*

Characteristic	N = 100	Percent (%)
Psychomotor Activity	38 (Normal)	38
	45 (Hyperactive)	45
	17 (Aggressive)	17
Patient Compliance	68 (Compliant)	68
	32 (Non-compliant)	32
Provisional Diagnosis	27 (BAD)	27
	48 (BP-1)	48
	25 (Manic)	25
Which Treatment Patient is Receiving	35 (Pharmacotherapy)	35
	5 (Psychotherapy)	5
	60 (Both)	60
Frequency of Hallucination	75 (Yes)	75
	25 (No)	25
Patient is Cooperative	69 (Yes)	69
	31 (No)	31
Previous Admitions in PIMS	13 (None)	13
	42 (Once)	42
	12 (Twice)	12
	30 (3-5Time)	30
	3 (Several Time)	3
Suicidal Thoughts	66 (Yes)	66
	34 (No)	34
Differential Diagnosis	41 (DIP)	41
	37 (SCZ)	37
	19 (Both)	19
	3 (Psychosis)	3

roles in the scientifically based treatment of BD, these preliminary findings, while not definitive, are suggestive and should be followed up. Both psychotherapy and pharmacotherapy is a practice and a point of view that has in effect been legislated without evidence. The strength of the preliminary findings poses a powerful challenge and invites serious investigation and further study. We hope that this study is only one in a series of long-awaited benefits to be realized by the development of unprecedentedly comprehensive data sets by the new national managed care industry that can be used to help us learn to take better and more cost-effective care of our patients.

6. ACKNOWLEDGEMENTS

The authors are thankful to their parents also for their moral support.

7. REFERENCES

1. Kupfer, D.J. The increasing medical burden in bipolar disorder. *Journal of American Medical Association* 293(20): 2528-2530 (2005).
2. Nurnberger, J.I., Jr. & T. Foroud. Genetics of bipolar affective disorder. *Current Psychiatry Report* 2(2): 147-157 (2000).
3. Potash, J.B., J. Toolan, J. Steele, E.B. Miller, J. Pearl, P.P. Zandi, T.G. Schulze, L. Kassem, S.G. Simpson, V. Lopez, D.F. MacKinnon & F.J. McMahon. The bipolar disorder phenome database: a resource for genetic studies. *American Journal of Psychiatry* 164(8): 1229-1237 (2007).
4. Sachs, G.S. & M.E.. Thase. Bipolar disorder therapeutics: maintenance treatment. *Biological Psychiatry* 48(6): 573-581 (2000).
5. Gogtay, N., A. Ordonez, D.H. Herman, K.M. Hayashi, D. Greenstein, C. Vaituzis, M. Lenane, L. Clasen, W. Sharp, J.N. Giedd, , D. Jung, T.F. Nugent Iii, W.A. Toga, E. Leibenluft, P.M. Thompson & J.L. Rapoport. Dynamic mapping of cortical development before and after the onset of pediatric bipolar illness. *Journal of Child Psychol Psychiatry* 48(9): 852-862 (2007).
6. Kessler, R.C., P. Berglund, O. Demler, R. Jin, K.R. Merikangas & E.E. Walters. Lifetime prevalence and age-of-onset distributions of DSM-IV disorders in the National Comorbidity Survey Replication. *Archieved of General Psychiatry* 62(6): 593-602 (2005).
7. Krishnan, K.R. Psychiatric and medical comorbidities of bipolar disorder. *Psychosomatic Medicine* 67(1): 1-8 (2005).
8. Schneck, C.D., D.J. Miklowitz, S. Miyahara, M. Araga, S. Wisniewski, L. Gyulai, M.H. Allen, M.E. Thase & G.S. Sachs. The prospective course of rapid-cycling bipolar disorder: findings from the STEP-BD. *American Journal of Psychiatry,* 165(3): 370-378 (2008).
9. Bizzarri, J., A. Sbrana, P. Rucci, L. Ravani, J. Massei, C. Gonnelli, S. Spagnolli, R. Doria, F. Raimondi, J. Endicott, L. Dell'Osso & G. Cassano. The spectrum of substance abuse in bipolar disorder: reasons for use, sensation seeking and substance sensitivity. *Bipolar Disorder* 9(3): 213-220 (2007).
10. Mueser, K.T., L.B. Goodman, S.L. Trumbetta, S.D. Rosenberg, C. Osher, R. Vidaver, P. Auciello & D.W. Foy. Trauma and posttraumatic stress disorder in severe mental illness. *Journal of Consulting and Clinical Psychology* 66(3): 493-499 (1998).
11. Calabrese, J.R., M.D. Shelton, D.J. Rapport, E.A. Youngstrom, K. Jackson, S. Bilali, S.J. Ganocy & R.L. Findling. A 20-month, double-blind, maintenance trial of lithium versus divalproex in rapid-cycling bipolar disorder.*Amweican Journal of Psychiatry* 162(11): 2152-2161 (2005).
12. Bowden, C.L., J.R. Calabrese, S.L. McElroy, L. Gyulai, A. Wassef, F. Petty, H.G. Pope, Jr., J.C. Chou, P.E. Keck, Jr., L.J. Rhodes, A.C. Swann, R.M. Hirschfeld, P.J. Wozniak & D.M.S. Group. A randomized, placebo-controlled 12-month trial of divalproex and lithium in treatment of outpatients with bipolar I disorder. *Archives of General Psychiatry* 57(5): 481-489 (2000).
13. Huxley, N.A., S.V. Parikh & R.J. Baldessarini. Effectiveness of psychosocial treatments in bipolar disorder: state of the evidence. *Harvard Review of Psychiatry* 8(3): 126-140 (2000).
14. Joffe, H., L.S. Cohen, T. Suppes, W.L. McLaughlin, P. Lavori, J.M. Adams, C.H. Hwang, J.E. Hall & G.S. Sachs. Valproate is associated with new-onset oligoamenorrhea with hyperandrogenism in women with bipolar disorder. *Biological Psychiatry* 59(11): 1078-1086 (2006).
15. Kupka, R.W., W.A. Nolen, R.M. Post, S.L. McElroy, L.L. Altshuler, K.D. Denicoff, M.A. Frye, P.E. Keck, Jr., G.S. Leverich, A.J. Rush, T. Suppes, C. Pollio & H.A. Drexhage. High rate of autoimmune thyroiditis in bipolar disorder: lack of association with lithium exposure. *Biological Psychiatry 51*(4): 305-311 (2002).
16. Miklowitz, D.J. A review of evidence-based psychosocial interventions for bipolar disorder. *Journal of Consult Clinical Psychology* 67(11): 28-33 (2006).
17. Sachs, G.S. & M.E. Thase. Bipolar disorder therapeutics: maintenance treatment. *Biol Psychiatry* 48(6): 573-581 (2000).
18. American Psychiatric Association. *Diagnostic and Statistical Manual of Mental Disorders,* 3rd ed rev. American Psychiatric Association, Washington, DC, USA (1987).
19. American Psychiatric Association. Practice guideline for the treatment of patients with bipolar disorder. *American Journal of Psychiatry* 12 (suppl): 1–36 (1994).
20. Rucci, P., E. Frank, B. Kostelnik, A. Fagiolini, A.G. Mallinger, H.A. Swartz & D.J. Kupfer. Suicide attempts in patients with bipolar I disorder during acute and maintenance phases of intensive treatment with pharmacotherapy and adjunctive psychotherapy. *American Journal of Psychiatry* 159(7): 1160-1164 (2002).
21. Pinquart, M., P. Duberstein, & J. Lyness. Treatments for later-life depressive conditions: a meta-analytic comparison of pharmacotherapy and psychotherapy. *American Journal of Psychiatry* 163(9): 1493-1501 (2006).
22. De Jonghe, F., M. Hendricksen, G. van Aalst, S. Kool, V. Peen, R. Van & J. Dekker. Psychotherapy alone and combined with pharmacotherapy in the treatment of depression. *The British Journal of Psychiatry* 185(1): 37-45 (2004).

Proceedings of the Pakistan Academy of Sciences 51 (3): 241–245 (2014)

ISSN: 0377 - 2969 (print), 2306 - 1448 (online)

Pakistan Academy of Sciences

Research Article

Concavity Solutions of Second-Order Differential Equations

Ibtisam Aldawish* and Maslina Darus

School of Mathematical Sciences, Faculty of Science and Technology,
Universiti Kebangsaan Malaysia, Bangi 43600,
Selangor Darul Ehsan, Malaysia

Abstract: In this article, we consider varieties of second-order linear differential equations in the unit disk. We show that the solutions of the second-order linear differential equations are concave univalent functions under some conditions.

Keywords: Analytic function, differential equation, concave function , univalent function

AMS Mathematics Subject Classification: 30C45

1. INTRODUCTION

Let A denote the class of functions normalized by

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \qquad (z \in \mathrm{D}), \tag{1}$$

which are analytic in the open unit disk $\mathrm{D} = \{z : |z| < 1\}$ on the complex plane $\mathbb{C}$. For functions $f \in A$ with $f'(z) \neq 0$ $(z \in \mathrm{D})$, we define the Schwarzian derivative of f by

$$S(f,z) = \left(\frac{f''(z)}{f'(z)}\right)' - \frac{1}{2}\left(\frac{f''(z)}{f'(z)}\right)^2, (f \in A;\ f'(z) \neq 0, z \in \mathbf{D}).$$

Let B_k denote the class of bounded functions $q(z) = q_1 z + q_2 z^2 + \ldots$ analytic in the unit disk D, for which $|q(z)| < K$. If $g(z) \in B_k$, then by using the Schwarz lemma [8], the function $q(z)$ defined by $q(z) = z^{-1/2} \int_0^z g(t)\, t^{-1/2} dt$

is also in B_k. Thus, in terms of derivatives , we have

$$\left|\frac{1}{2} q(z) + zq'(z)\right| < K \Rightarrow |q(z)| < K, (z \in \mathrm{D}). \tag{2}$$

If we let

$$\psi(u,v) = \frac{1}{2}u + v.$$

We can write (2) as

$$|\psi(q(z), zq'(z))| < K \Rightarrow |q(z)| < K. \tag{3}$$

Saitoh [11] and Millar [7] showed that (3) holds true for functions $\psi(u,v)$ in the class H_k given by Definition 1.1. below.

Definition 1.1 (see [7]) Let H_k be the set of complex functions $\psi(u,v)$ satisfying the following conditions:

i. $\psi(u,v)$ is continuous in a domain $\mathrm{D} \subset \mathbf{C} \times \mathbf{C}$;
ii. $(0,0) \in \mathrm{D}$ and $|\psi(0,0)| < K$;
iii. $|\psi(Ke^{i\theta}, Te^{i\theta})| \geq K$ when $(Ke^{i\theta}, Te^{i\theta}) \in \mathrm{D}$, θ is real and $T \geq K$.

Definition 1.2 (see [6]) Let $\psi \in H_k$ with corresponding domain D. We denote by $B_k(\psi)$ those functions $q(z) = q_1 z + q_2 z^2 + \ldots$ which are analytic in D satisfying :

i. $(q(z), zq'(z)) \in \mathrm{D}$,

Received, February 2014; Accepted, August 2014
*Corresponding author: Maslina Darus; Email: maslina@ukm.edu.my

ii. $|\psi(q(z), zq'(z))| < K$ $(z \in \mathbf{D})$.

Many other authors also studied the geometric properties solutions of a class of second-order linear differential equations, for example one can refer to [1, 4, 6, 7, 10, 11, 12].

We now state the following result due to Miller [7].

Theorem 1.3 (Miller [7]) Let $p(z)$ be an analytic function in the unit disk D with $|zp(z)| < 1$. Let $v(z), z \in \mathbf{D}$, be the unique solution of

$$v''(z) + p(z)v(z) = 0,$$

with $v(0) = 0$ and $v'(0) = 1$. Then, $\left|\frac{zv'(z)}{v(z)} - 1\right| < 1$ and $v(z)$ is a starlike conformal map of the unit disk D.

Theorem 1.3 is related rather closely to some earlier results of Robertson [10] and Nehari [8], which we recall Theorem 1.4 and Theorem 1.5, respectively, as follows:

Theorem 1.4 (Robertson [10]) Let $zp(z)$ be an analytic function in D and $\Re\{z^2 p(z)\} \le \frac{\pi^2}{4}|z|^2$ $(z \in \mathbf{D})$. Then, the unique solution $v = v(z)$ of the following initial-value problem:

$$v''(z) + p(z)v(z) = 0 \quad (v(0) = 0, v'(0) = 1)$$

is univalent and starlike in D. The constant $\pi^2/4$ is the best possible one.

Theorem 1.5 (Nehari [8]) If $f(z) \in A$ and it satisfies $|S(f, z)| \le \frac{\pi^2}{2}$ $(z \in \mathbf{D})$, then $f(z)$ is univalent.

The next theorems, which are due to Saitoh [11,12] and Owa et al. [9], involve several geometric properties of the solutions of the second-order linear differential equations.

Theorem 1.6 (Saitoh [11]) Let $a(z)$ and $b(z)$ be analytic in D with $\left|z\left(b(z) - \frac{1}{2}a'(z) - \frac{1}{4}[a(z)]^2\right)\right| < \frac{1}{2}$ and $|a(z)| < 1$. Let $v(z)$ $(z \in \mathbf{D})$ be the solution of the following second order linear differential equation

$$v''(z) + a(z)v'(z) + b(z)v(z) = 0, \; v(0) = 0, v'(0) = 1.$$

Then, $v(z)$ is starlike in D.

Theorem 1.7 (Owa et al [9]) Let the function $a(z)$ and $b(z)$ be analytic in D with $\Re\{za(z)\} > -2K$ and $\left|z^2\left(b(z) - \frac{1}{2}a'(z) - \frac{1}{4}[a(z)]^2\right)\right| < K$. Also, let $v(z)$ denote the solution of the initial- value problem equation:

$$v''(z) + a(z)v'(z) + b(z)v(z) = 0,$$
$$v(0) = 0, v'(0) = 1.$$

Then,

$$1 - K - \frac{1}{2}\Re\{za(z)\} < \Re\left(\frac{zv'(z)}{v(z)}\right) <$$
$$1 + K - \frac{1}{2}\Re\{za(z)\}, (z \in \mathbf{D}; K > 0).$$

Theorem 1.8 (Saitoh [12]) Let $p_n(z)$ be the non-constant polynomial of degree $n \ge 1$ with $|p_n(z)| < K$ $(z \in \mathbf{D};\ K > 0)$. Let $v(z)$ be the solution of the initial-value problem:

$$v''(z) + p_n(z)v(z) = 0, \quad v(0)=0;\ v'(0) = 1.$$

Then, we have

$$1 - K < \Re\left\{\frac{zv'(z)}{v(z)}\right\} < 1 + K \quad (z \in \mathbf{D}).$$

The following theorem was proved by Abubaker and Darus [1] using the third-order linear differential equation.

Theorem 1.9 (Abubaker & Darus [1]) Let $Q(z) = \sum_{n=0}^{\infty} b_n z^n$ be analytic in D with $\sum_{n=0}^{\infty} |b_n| < K$ $(z \in \mathbf{D};\ K > 0)$, and let $v(z)$ denote the solution of the initial-value problem

$$v'''(z) + Q(z)v'(z) = 0, \quad z \in \mathbf{D}.$$

Then,

$$1 - K < \Re\left\{1 + \frac{zv''(z)}{v'(z)}\right\} < 1 + K \quad (z \in \mathbf{D};\ K > 0).$$

Next, we state the family of concave functions which is our main focus here.

A function $f:\mathbf{D}\to\mathbb{C}$ is said to belong to the family $C_0(\alpha)$ if f satisfies the following conditions:

i. f is analytic in D with the standard normalization $f(0)=f'(0)-1=0$. In addition it satisfies $f(1)=\infty$.

ii. f maps D conformally onto a set whose complement with respect to C is convex.

iii. The opening angle of $f(\mathbf{D})$ at ∞ is less than or equal $\pi\alpha, \alpha\in(1,2]$.

The class $C_0(\alpha)$ is referred to as the class of concave univalent functions and for a detailed discussion about concave functions, we refer to [2, 3, 5].

We recall the analytic characterization for functions f in $C_0(\alpha), \alpha\in(1,2]: f\in C_0(\alpha)$ if and only if $\Re P_f(z)>0,\ \ z\in\mathbf{D}$, where

$$p_f(z)=\frac{2}{\alpha-1}\left[\frac{\alpha+1}{2}\frac{1+z}{1-z}-1-z\frac{f''(z)}{f'(z)}\right].$$

Before we establish our main results, we need to indicate to the following theorems to prove our results.

Theorem 1.10 (see [11]) For any $\psi\in H_k, B_k(\psi)\subset B_k, (\psi\in H_k; K>0)$.

Theorem 1.10 leads us to immediately to the following result, which was also given by Saitoh [11].

Theorem 1.11 (see [11]) Let $\psi\in H_k$ and $b(z)$ be an analytic function in D with $|b(z)|<K$. If the differential equation

$$\psi(q(z), zq'(z))=b(z), q(0)=0, q'(0)=1$$

has a solution $q(z)$ analytic in D, then $|q(z)|<K$.

The objective of the present paper is to investigate the concavity of solutions of the second-order linear differential equations.

2. MAIN RESULTS

We derive the following results by employing Theorem 1.11. First, we concentrate on the concavity of the solution of the following initial-value problem:

$$q''(z)+a(z)q'(z)+b(z)q(z)=0. \tag{4}$$

Theorem 2.1 Let $a(z), b(z)$ be analytic functions in D such that

$$\left|z^2b(z)\right|<K,\quad (z\in \mathrm{D}; K>0). \tag{5}$$

Let $q(z), z\in\mathbf{D}$ be the solution of the initial value problem (4) in D. Then,

$$\frac{2}{\alpha-1}\left(\frac{\alpha+1}{2}-k\right)<$$
$$\Re\left\{\frac{2}{\alpha-1}\left(\frac{\alpha+1}{2}\frac{1+z}{1-z}-\frac{zq'(z)}{q(z)}\right)\right\}<$$
$$\frac{20}{\alpha-1}(\alpha+K+1), \tag{6}$$

where $\alpha\in(1,2]$.

Proof. We recall $f\in C_0(\alpha)$ if and only if $\Re P_f(z)>0$ in D, where

$$p_f(z)=\frac{2}{\alpha-1}\left(\frac{\alpha+1}{2}\frac{1+z}{1-z}-\frac{zg'(z)}{g(z)}\right)$$

with $g(z)=zf'(z)$. We note that p is analytic in D with $p(0)=1$.

If we set

$$r(z)=-\frac{zq'(z)}{q(z)}\qquad (z\in\mathbf{D}) \tag{7}$$

then, $r(z)$ is analytic in D, $r(0)=0$ and (4) becomes

$$\left(r(z)\right)^2+(1-za(z))r(z)-zr'(z)=-z^2b(z). \tag{8}$$

Thus (8) can be rewritten as

$$\psi(r(z), zr'(z))=-z^2b(z),$$

where $\psi(s,t)=s^2+(1-za(z))s-t$.

Since

i. $\psi(s,t)$ is continuous in a domain $\mathbf{D}\subset\mathbf{C}\times\mathbf{C}$;

ii. $(0,0)\in \mathbf{D}$ and $|\psi(0,0)| = 0 < K$;

iii. For $(Ke^{i\theta}, Te^{i\theta}) \in \mathbf{D}$, θ is real and $T \geq K, |\psi(Ke^{i\theta}, Te^{i\theta})| = |K^2 e^{i\theta} + K - T| > T \geq K$.

We conclude that $\psi(s,t) \in H_k$.

From the hypothesis (5) and by employing Theorem 1.11 , we obtain that

$|r(z)| < K, \quad K > 0$.

Combine this with (7) we have

$$\left| -\frac{zq''(z)}{q'(z)} \right| < K, \quad K > 0.$$

This leads to the following relations

$$\Re\left\{ \frac{2}{\alpha-1}\left(\frac{\alpha+1}{2}\frac{1+z}{1-z} - K \right) \right\} <$$
$$\Re\left\{ \frac{2}{\alpha-1}\left(\frac{\alpha+1}{2}\frac{1+z}{1-z} - \frac{zq'(z)}{q(z)} \right) \right\} <$$
$$\Re\left\{ \frac{2}{\alpha-1}\left(\frac{\alpha+1}{2}\frac{1+z}{1-z} + K \right) \right\}.$$

We find that

$$\frac{2}{\alpha-1}\left(\frac{\alpha+1}{2} - K \right) < \Re\left\{ \frac{2}{\alpha-1}\left(\frac{\alpha+1}{2}\frac{1+z}{1-z} - K \right) \right\}$$

and

$$\Re\left\{ \frac{2}{\alpha-1}\left(\frac{\alpha+1}{2}\frac{1+z}{1-z} + K \right) \right\} < \frac{2}{\alpha-1}\left(20\frac{\alpha+1}{2} + K \right).$$

We can simplify the last expressions and obtain (6). This completes the proof of the theorem.

If we take $K < \frac{\alpha+1}{2}$ in Theorem 2.1, then we deduce the following corollary.

Corollary 2.2 Let $a(z), b(z)$ be analytic functiosn in D such that $|z^2 b(z)| < \frac{\alpha+1}{2}, (z \in \mathbf{D}; \alpha \in (1,2])$. Let $q(z)$ be the solution of the initial –value problem (4). Then, $q(z) \in C_0(\alpha)$.

Example 2.3 Let $a(z) = 0$ and $b(z) = 1$ in Corollary 2.2 . Then, for $z \to 1$ and $\alpha = 2$, the solution of the following initial-value problem :

$$q''(z) + q(z) = 0, \quad q(0) = 0, q'(0) = 1$$

is given by

$q(z) = \sin z \in C_0(2)$.

We next show that the following differential equation

$$q''(z) + M(z)q(z) = 0 \tag{9}$$

has a solution $q(z)$, which is concave univalent in D.

Theorem 2.4 Let $M(z)$ be analytic functions in D such that

$$|z^2 M(z)| < K \quad (z \in \mathbf{D}, K > 0). \tag{10}$$

Let $q(z), z \in D$ be the solution of the initial value problem (9) . Then,

$$\frac{2}{\alpha-1}\left(\frac{\alpha+1}{2} - K \right) <$$
$$\Re\left\{ \frac{2}{\alpha-1}\left(\frac{\alpha+1}{2}\frac{1+z}{1-z} - \frac{zq'(z)}{q(z)} \right) \right\} <$$
$$\frac{20}{\alpha-1}(\alpha + K + 1), \tag{11}$$

where $\alpha \in (1,2]$.

Proof. If we put

$$r(z) = -\frac{zq'(z)}{q(z)} \qquad (z \in \mathbf{D}), \tag{12}$$

we see that $r(z)$ is analytic in D, $r(0) = 0$ and (9) becomes

$$(r(z))^2 + r(z) - zr'(z) = -z^2 M(z). \tag{13}$$

We can write this equality as

$$\psi(r(z), zr'(z)) = -z^2 M(z),$$

where $\psi(s,t) = s^2 + s - t$.

It is easy to check that the conditions of Definition 1.1 are satisfied.

Therefore from (12) and in order to apply Theorem 1.11, we obtain

$|r(z)| < K, \quad K > 0,$

which implies that

$$\left| -\frac{zq'(z)}{q(z)} \right| < K, \quad K > 0.$$

Hence we conclude that

$$\frac{2}{\alpha-1}\left(\frac{\alpha+1}{2}-K\right)<\Re\left\{\frac{2}{\alpha-1}\left(\frac{\alpha+1}{2}\frac{1+z}{1-z}-\frac{zq'(z)}{q(z)}\right)\right\}<$$

$$\frac{20}{\alpha-1}(\alpha+K+1)$$

$(z\in \mathrm{D};\ \mathrm{K}>0;\ \alpha\in(1,2])$.

Thus, the proof is complete.

Next we obtain the Corollary by following substituting $K<\frac{\alpha+1}{2}$ in Theorem 2.4.

Corollary 2.5 Let $M(z)$ be analytic functions in D such that $\left|z^2M(z)\right|<\frac{\alpha+1}{2}$, $(z\in \mathbf{D};\ \alpha\in(1,2])$. Let $q(z)$ be the solution of the initial –value problem (9) . Then, $q(z)\in C_0(\alpha)$.

3. CONCLUSIONS

The varieties of second-order linear differential equations in the unit disk are discussed. Moreover, we showed that the solutions of the second-order linear differential equations are concave univalent functions under some conditions.

4. ACKNOWLMEEDGNTS

The work reported in the article was supported by UKM's grant: AP-2013-009 and DIP-2013-001.

5. REFERENCES

1. Abubaker, A.A. & M. Darus. Geometric properties solutions of a class of third-order linear differential equations. *ISRN Applied Mathematics*, Article ID 249853, 5 pp. (2011).
2. Avkhadiev, F.G., C. Pommerenke & K-J. Wirths. Sharp inequalities for the coefficient of concave schlicht functions. *Commentarii Mathematici Helvetici* 81: 801-807 (2006).
3. Avkhadiev, F.G. & K-J. Wirths. Concave schlicht functions with bounded opening angle at infinity. *Lobachevskii Journal of Mathematics* 17: 3-10 (2005).
4. Boyce, W.E. Elementary differential equations. *John Willey & Sons,* 212 pp. (1977).
5. Cruz, L. & C. Pommerenke. On concave univalent functions. *Complex Variables and Elliptic Equations* 52: 153-159 (2007).
6. Miller, S.S. & P.T. Mocanu. Second-order differential inequalities in the complex plane. *Journal of Mathematical Analysis and Applications* 65: 289-305 (1978).
7. Miller, S.S. A class of differential inequalities implying boundedness. *Illinois Journal of Mathematics* 20: 647-649 (1976).
8. Nehari, Z. The Schwarzian derivative and schlicht functions. *Bulletin of the American Mathematical Society* 55 : 545-551 (1949).
9. Owa, S., H. Saitoh, H.M. Srivastava & R. Yamakawa. Geometric properties of solutions of a class of differential equations. *Computers & Mathematics with Applications* 47: 1689-1696 (2004).
10. Robertson, M.S. Schlicht solutions of $w''+pw=0$. *Transactions of the American Mathematical Society* 76: 254-274(1954).
11. Saitoh, H. Univalence and starlikeness of solutions of $w''+aw'+bw=0$. *Annales Universitatis Mariae Curie Skłodowska. Sectio A*. 53 : 209-216(1999).
12. Saitoh, H. Geometric properties of solutions of a class of ordinary linear differential equations. *Applied Mathematics and Computation* 187: 408-416 (2007).

Proceedings of the Pakistan Academy of Sciences 51 (3): 247–257 (2014)

ISSN: 0377 - 2969 (print), 2306 - 1448 (online)

Pakistan Academy of Sciences

Research Article

Generalized Ostrowski Type Inequalities on Time Scales with Applications

Sabir Hussain[1] and Khurrem Shehzad[2*]

[1]Department of Mathematics, University of Engineering and Technology, Lahore, Pakistan
[2]Department of Mathematics, Virtual University of Pakistan, Lahore, Pakistan

Abstract: Generalized Ostrowski type inequalities for function of three independent variables on time scales are derived that generalized some existing and classical inequalities with some applications for generalized polynomials.

Keywords: Ostrowski inequality; generalized polynomial; Time scales; rd-continuous function

1. INTRODUCTION

The role of mathematical inequalities within the mathematical branches as well as in its enormous applications should not be underestimated. The appearance of the new mathematical inequality often puts on firm foundation for the heuristic algorithms and procedures used in applied sciences. Among others one of the main inequality, which stipulates a bound between a function evaluated at an interior point x and the integral mean of f over an interval, called Ostrowski inequality is defined as [9]:

$$\left|f(x)-\frac{1}{b-a}\int_a^b f(t)dt\right| \leq \sup_{a<x<b}|f'(x)|\left[\frac{1}{4}+\left(\frac{x-\frac{a+b}{2}}{b-a}\right)^2\right](b-a), \qquad (1)$$

where $f:[a,b]\to \boldsymbol{R}$ is a differentiable function. The constant factor $\frac{1}{4}$ is the best possible one. It has a lot of applications in numerical analysis, probability theory and in special means. P. Cerone et al. [3] proved an Ostrowski type inequality for n-times differentiable function. For other similar results for n-times differentiable functions, see [1, 2, 4, 5]. Sofo determined an Ostrowski type inequality in three independent variables [14]. For more results about Ostrowski consults various prior publications [7, 10, 11, 15, 16, 17, 18, 19, 20, 21].

Our main purpose in this paper is to prove some new results related to an Ostrowski inequality in three variables on time scales, generalizing some existing and classical results. Some applications to generalized polynomial are also given.

In Section 2, some time scales essentials are given. In Section 3, some new results are given and in the last Section 4, some applications for generalized polynomials are given.

2. TIME SCALES ESSENTIALS

A time scale (or measure chain) is a non-empty closed subset of the real $\boldsymbol{R}$, together with the topology of subspace of $\boldsymbol{R}$ and we usually denote it by the symbol $\mathbb{T}$. The two most popular examples are $\mathbb{T}=\boldsymbol{R}$ and $\mathbb{T}=\mathbb{Z}$. For any interval $\boldsymbol{I}$ of $\boldsymbol{R}$ (open or closed) $\boldsymbol{I}_{\mathbb{T}}=\boldsymbol{I}\cap\mathbb{T}$ is called a time scales interval. We define the forward and backward jump operators $\sigma,\rho:\mathbb{T}\to\mathbb{T}$ by:

Received, February 2014; Accepted, August 2014
*Corresponding author: Khurrem Shehzad; Email: khurremshehzad@gmail.com

$$\sigma(t) = \inf\{s \in \mathbb{T} : s > t\} \text{and} \rho(t) = \sup\{s \in \mathbb{T} : s < t\},$$

(supplemented by $\sup\mathbb{T} = \inf\phi$ and $\sup\phi = \inf\mathbb{T}$ where ϕ denotes the empty set).The set $\mathbb{T}^k$is defined to be $\mathbb{T}$ if $\mathbb{T}$ does not have a left scattered maximum; otherwiseit is $\mathbb{T}$ without this left scattered maximum. The graininess $\mu: \mathbb{T} \to [0, \infty)$ defined by

$$\mu(t) = \sigma(t) - t.$$

Hence the graininess function is constant 0 if $\mathbb{T} = \mathbb{R}$ while it is constant 1 if $\mathbb{T} = \mathbb{Z}$.

However, a time scale $\mathbb{T}$ could have nonconstant graininess. Let $f: \mathbb{T} \to \mathbf{R}$, be a function then $f^{\sigma}: \mathbb{T} \to \mathbf{R}$ defined by $f^{\sigma}(t) = f(\sigma(t))$ for $t \in \mathbb{T}$,where $\sigma(t)$ is defined above. We also, say that f is delta differentiable (or simply: differentiable) at $t \in \mathbb{T}^k$ provided there exists an α such that for all $\epsilon > 0$ there is a neighborhood $\aleph$ of t with

$$\left|\left[f\left(\sigma(t)\right) - f(s)\right] - \alpha[\sigma(t) - s]\right| \leq \epsilon|\sigma(t) - s| \qquad \text{for all } s \in \aleph.$$

In this case we denote the α by$f^{\Delta}(t)$, and if f is differentiable for every $t \in \mathbb{T}^k$, then f is said to be differentiable on $\mathbb{T}$ and f^{Δ} is a new function on $\mathbb{T}^k$. If f is differentiable at $t \in \mathbb{T}^k$, then it is easy to see that

$$f^{\Delta}(t) = \begin{cases} \lim\limits_{s \to t (s \in \mathbb{T})} \dfrac{f(t) - f(s)}{t - s} & if \;\; \mu(t) = 0 \\ \dfrac{f(\sigma(t)) - f(t)}{\mu(t)} & if \;\; \mu(t) > 0 \end{cases}$$

Several useful delta derivative formulae can be recorded in [13, Lemma 1,2] (see also [12]).

A continuous function $f: \mathbb{T} \to \boldsymbol{R}$is said to be pre-differentiable with $D \subset \mathbb{T}^k$ (where $\mathbb{T}^k \backslash D$ is countable) as differentiation region, contains no right scattered elements of $\mathbb{T}$ and f is differentiable at $t \in D$. A function f is called regulated if its right sided limit exist at all right dense points in $\mathbb{T}$ and its left sided limit exist at all left dense points in $\mathbb{T}$.If there exist a pre-differentiable function F such that

$$F^{\Delta}(t) = f(t) \quad \forall\, t \in D$$

Then F is called pre-antiderivative of f and indefinite integral of f is defined by

$$\int f(t)\Delta t = F(t) + C$$

Where C is a constant and

$$\int_s^t f(\tau)\Delta\tau = F(t) - F(s), \qquad \text{for } s, t \in \mathbb{T}.$$

A function $f: \mathbb{T} \to \mathbf{R}$ is said to be rd-continuous, provided it is continuous at every right-dense point and its left sided limit exists at every left dense pointin $\mathbb{T}$. We denote the set of functions $f: \mathbb{T} \to \mathbf{R}$ whose nth order derivative is rd-continuous by $C_{rd}^n(\mathbb{T}, \boldsymbol{R}), n \in \mathbb{N}$. The importance of rd-continuous functions is revealed by the following existence result by Hilger [8]. Every rd-continuous function possesses an antiderivative.

The generalized polynomial is the function $h_k : \mathbb{T}^2 \to \mathbf{R}$, $k \in \mathbb{N}_0$, defined recursively as follows $h_0(t, s) = 1$ for all $s, t \in \mathbb{T}$, and given $h_k$fork $\in \mathbb{N}_0$, the functionh_{k+1} is given by:

$$h_{k+1}(t, s) = \int_s^t h_k(\tau, s)\, \Delta\tau \qquad \text{for all } s, t \in \mathbb{T}.$$

3. MAIN RESULTS

To make the presentation compact and easier to understand, we make some symbolical representations. Here, $n, m, q \in \mathbb{N}$and k a non-negative integer.

$$P_n(x,r)=\begin{cases}h_n(r,a_1), & a_1\le r\le x\\ h_n(r,b_1), & x<r\le b_1\end{cases}.\quad Q_m(y,s)=\begin{cases}h_m(s,a_2), & a_2\le s\le y\\ h_m(s,b_2), & y<s\le b_2\end{cases}.$$

$$S_q(z,t)=\begin{cases}h_q(t,a_3), & a_3\le t\le z\\ h_q(t,b_3), & z<t\le b_3\end{cases}.\quad w_{m,k}(x)=X_m(x)f^{\Delta^{m-1}}\left(\sigma^k(x)\right).$$

$$X_{k+1}(x)=(-1)^k h_{k+1}(x,a_1)+(-1)^{k+1}h_{k+1}(x,b_1)$$
$$Y_{k+1}(y)=(-1)^k h_{k+1}(y,a_2)+(-1)^{k+1}h_{k+1}(y,b_2)$$
$$Z_{k+1}(z)=(-1)^k h_{k+1}(z,a_3)+(-1)^{k+1}h_{k+1}(z,b_3)$$

Lemma 1. Let $f:[a_1,b_1]_{\mathbb{T}}\to \boldsymbol{R}$be a function such that $f\in C_{rd}^k([a_1,b_1]_{\mathbb{T}},\boldsymbol{R}), 0\le k\le n-1$. If graininess, μ, is constant. Then, for $x\in[a_1,b_1]$, the following identity holds:

$$\int_{a_1}^{b_1} f\left(\sigma^n(t)\right)\Delta t-\sum_{k=0}^{n-1}X_{n-k}(x)\,f^{\Delta^{n-k-1}}\left(\sigma^k(x)\right)=(-1)^n\int_{a_1}^{b_1}P_n(x,t)\,f^{\Delta^n}(t)\Delta t \tag{2}$$

Proof. Consider

$$I_{n,0}(x)=(-1)^n J_{n,0}(a_1,x,b_1)=(-1)^n\int_{a_1}^{b_1}P_n(x,t)\,f^{\Delta^n}\left(\sigma^0(t)\right)\Delta t.$$

$$J_{n,0}(a_1,x,x)=\int_{a_1}^{x}h_n(t,a_1)\,f^{\Delta^n}\left(\sigma^0(t)\right)\Delta t=\left|h_n(t,a_1)f^{\Delta^{n-1}}\left(\sigma^0(t)\right)\right|_{a_1}^{x}-\int_{a_1}^{x}h_n(t,a_1)\,f^{\Delta^{n-1}}\left(\sigma^1(t)\right)\Delta t$$
$$=h_n(x,a_1)f^{\Delta^{n-1}}\left(\sigma^0(x)\right)-J_{n-1,1}(a_1,x,x)(3)$$

Similarly:

$$J_{n,0}(x,x,b_1)=\int_{x}^{b_1}h_n(t,b_1)\,f^{\Delta^n}\left(\sigma^0(t)\right)\Delta t=-h_n(x,b_1)f^{\Delta^{n-1}}\left(\sigma^0(x)\right)-J_{n-1,1}(x,x,b_1) \tag{4}$$

Addition of (3) and (4) yields:

$$I_{n,0}(x)-I_{n-1,1}(x)=-w_{n,0}(x). \tag{5}$$

Similarly:

$$I_{n-1,1}(x)-I_{n-2,2}(x)=-w_{n-1,1}(x)$$
$$\vdots$$
$$I_{1,n-1}(x)-I_{0,n}(x)=-w_{1,n-1}(x).$$

Addition of these relations yields:

$$I_{n,0}(x)=I_{0,n}(x)-\sum_{k=0}^{n-1}w_{n-k,k}(x).$$

And hence (2) is proved.

Remark 1. The above identity (2) generalizes the identity in [6]. In particular for $\mathbb{T}=\boldsymbol{R}$ it is re-captured. Moreover by using the properties of modulus and supremum norm, for $\mathbb{T}=\boldsymbol{R}$,we get [6, Theorem 1] and for $n=1$ we recapture the classical Ostrowski inequality (1).

Theorem 1. Let $a_i,b_i\in\mathbb{T}, 1\le i\le 3$; let $g:\prod_{i=1}^3[a_i,b_i]_{\mathbb{T}}\to\boldsymbol{R}$be rd-continuous function such that the partial derivatives $\frac{\partial^{k+l+p}g(.,.,)}{\Delta x^k\Delta y^l\Delta z^p}, 0\le k\le n-1; 0\le l\le m-1; 0\le p\le q-1$ exist and are continuous on $\prod_{i=1}^3[a_i,b_i]_{\mathbb{T}}$.Then, for $(x,y,z)\in\Pi_{i=1}^3[a_i,b_i]$, the following inequality holds:

$$\left|\int_{a_1}^{b_1}\int_{a_2}^{b_2}\int_{a_3}^{b_3} g\big(\sigma^n(r),\sigma^m(s),\sigma^q(t)\big)\Delta t\Delta s\Delta r-\sum_{k=0}^{n-1}\sum_{l=0}^{m-1}\sum_{p=0}^{q-1} X_{k+1}(x)Y_{l+1}(y)Z_{p+1}(z)\right.$$

$$\times\frac{\partial^{k+l+p} g\left(\sigma^{n-k-1}(x),\sigma^m(y),\sigma^q(z)\right)}{\Delta x^k\Delta y^l\Delta z^p}+(-1)^{q+1}\sum_{k=0}^{n-1}\sum_{l=0}^{m-1} X_{k+1}(x)Y_{l+1}(y)\int_{a_3}^{b_3} S_q(z,t)$$

$$\times\frac{\partial^{k+l+q} g\left(\sigma^{n-k-1}(x),\sigma^m(y),\sigma^q(t)\right)}{\Delta x^k\Delta y^l\Delta t^q}\Delta t+(-1)^{m+1}\sum_{k=0}^{n-1}\sum_{p=0}^{q-1} X_{k+1}(x)Z_{p+1}(z)\int_{a_2}^{b_2} Q_m(y,s)$$

$$\times\frac{\partial^{k+m+p} g\left(\sigma^{n-k-1}(x),\sigma^m(s),\sigma^q(z)\right)}{\Delta x^k\Delta s^m\Delta z^p}\Delta s+(-1)^{m+q+1}\sum_{k=0}^{n-1} X_{k+1}(x)$$

$$\times\int_{a_2}^{b_2}\int_{a_3}^{b_3} Q_m(y,s)S_q(z,t)\frac{\partial^{k+m+q} g\left(\sigma^{n-k-1}(x),\sigma^m(s),\sigma^q(t)\right)}{\Delta x^k\Delta s^m\Delta t^q}\Delta t\Delta s+(-1)^{n+1}$$

$$\times\sum_{l=0}^{m-1}\sum_{p=0}^{q-1} Y_{l+1}(y)Z_{p+1}(z)\int_{a_1}^{b_1} P_n(x,r)\frac{\partial^{n+l+p} g\big(r,\sigma^m(y),\sigma^q(z)\big)}{\Delta r^n\Delta y^l\Delta z^p}\Delta r+(-1)^{n+q+1}$$

$$\times\sum_{l=0}^{m-1} Y_{l+1}(y)\int_{a_1}^{b_1}\int_{a_3}^{b_3} P_n(x,r)S_q(z,t)\frac{\partial^{n+l+q} g\big(r,\sigma^m(y),\sigma^q(t)\big)}{\Delta r^n\Delta y^l\Delta t^q}\Delta t\Delta r+(-1)^{m+n+1}$$

$$\left.\times\sum_{p=0}^{q-1} Z_{P+1}(z)\int_{a_1}^{b_1}\int_{a_2}^{b_2} P_n(x,r)Q_m(y,s)\frac{\partial^{n+m+p} g\big(r,\sigma^m(s),\sigma^q(z)\big)}{\Delta r^n\Delta s^m\Delta z^p}\Delta s\Delta r\right|$$

$$\leq\left\|\frac{\partial^{n+m+q} g}{\Delta r^n\Delta s^m\Delta t^q}\right\|_\infty\int_{a_1}^{b_1}\int_{a_2}^{b_2}\int_{a_3}^{b_3}\big|P_n(x,r)Q_m(y,s)S_q(z,t)\big|\Delta t\Delta s\Delta r. \tag{6}$$

Proof. Consider the following identity by lemma 1:

$$\int_{a_1}^{b_1} f\big(\sigma^n(r)\big)\Delta r=\sum_{k=0}^{n-1} X_{k+1}(x)\,f^{\Delta^k}\left(\sigma^{n-k-1}(x)\right)+(-1)^n\int_{a_1}^{b_1} P_n(x,r)f^{\Delta^n}(r)\Delta r. \tag{7}$$

For the partial mapping $g\big(.,\sigma^m(s),\sigma^q(t)\big)$ for $(s,t)\in\prod_{i=2}^{3}[a_i,b_i]$

$$\int_{a_1}^{b_1} g\big(\sigma^n(r),\sigma^m(s),\sigma^q(t)\big)\Delta r=\sum_{k=0}^{n-1} X_{k+1}(x)\frac{\partial^k g\left(\sigma^{n-k-1}(x),\sigma^m(s),\sigma^q(t)\right)}{\Delta x^k}$$

$$+(-1)^n\int_{a_1}^{b_1} P_n(x,r)\frac{\partial^n g\big(r,\sigma^m(s),\sigma^q(t)\big)}{\Delta r^n}\Delta r. \tag{8}$$

Δ −integrating over $s\in[a_2,b_2]$

$$\int_{a_1}^{b_1}\int_{a_2}^{b_2} g\big(\sigma^n(r),\sigma^m(s),\sigma^q(t)\big)\Delta s\Delta r=\sum_{k=0}^{n-1} X_{k+1}(x)\int_{a_2}^{b_2}\frac{\partial^k g\left(\sigma^{n-k-1}(x),\sigma^m(s),\sigma^q(t)\right)}{\Delta x^k}\Delta s$$

$$+(-1)^n\int_{a_1}^{b_1}\int_{a_2}^{b_2} P_n(x,r)\frac{\partial^n g\big(r,\sigma^m(s),\sigma^q(t)\big)}{\Delta r^n}\Delta s\Delta r. \tag{9}$$

For the partial mapping $\frac{\partial^k g\left(\sigma^{n-k-1}(x),\ .\ ,\sigma^q(t)\right)}{\Delta x^k}$ on $[a_2, b_2]$ for $(x,t) \in [a_1, b_1] \times [a_3, b_3]$

$$\int_{a_2}^{b_2} \frac{\partial^k g\left(\sigma^{n-k-1}(x), \sigma^m(s), \sigma^q(t)\right)}{\Delta x^k} \Delta s = \sum_{l=0}^{m-1} \frac{\partial^{k+1} g\left(\sigma^{n-k-1}(x), \sigma^m(y), \sigma^q(t)\right)}{\Delta x^k \Delta y^l}$$

$$\times Y_{l+1}(y) + (-1)^m \int_{a_2}^{b_2} Q_m(y,s) \frac{\partial^{k+m} g\left(\sigma^{n-k-1}(x), \sigma^m(s), \sigma^q(t)\right)}{\Delta x^k \Delta s^m} \Delta s. \tag{10}$$

Similarly:

$$\int_{a_2}^{b_2} \frac{\partial^n g\left(r, \sigma^m(s), \sigma^q(t)\right)}{\Delta r} \Delta s = \sum_{l=0}^{m-1} \frac{\partial^{n+1} g\left(r, \sigma^m(y), \sigma^q(t)\right)}{\Delta r^n \Delta y^l} Y_{l+1}(y)$$

$$+(-1)^m \int_{a_2}^{b_2} Q_m(y,s) \frac{\partial^{n+m} g\left(r, \sigma^m(s), \sigma^q(t)\right)}{\Delta r^n \Delta s^m} \Delta s. \tag{11}$$

From (9) – (11)

$$\int_{a_1}^{b_1} \int_{a_2}^{b_2} g\left(\sigma^n(r), \sigma^m(s), \sigma^q(t)\right) \Delta s \Delta r$$

$$= \sum_{k=0}^{n-1} \sum_{l=0}^{m-1} X_{k+1}(x) Y_{l+1}(y) \frac{\partial^{k+1} g\left(\sigma^{n-k-1}(x), \sigma^m(y), \sigma^q(t)\right)}{\Delta x^k \Delta y^l}$$

$$+ (-1)^m \sum_{k=0}^{n-1} X_{k+1}(x) \int_{a_2}^{b_2} Q_m(y,s) \frac{\partial^{k+m} g\left(\sigma^{n-k-1}(x), \sigma^m(s), \sigma^q(t)\right)}{\Delta x^k \Delta s^m} \Delta s$$

$$+ (-1)^n \sum_{l=0}^{m-1} Y_{l+1}(y) \int_{a_1}^{b_1} P_n(x,r) \frac{\partial^{n+l} g\left(r, \sigma^m(y), \sigma^q(t)\right)}{\Delta r^n \Delta y^l} \Delta r$$

$$+ (-1)^{m+n} \int_{a_1}^{b_1} \int_{a_2}^{b_2} P_n(x,r) Q_m(y,s) \frac{\partial^{n+m} g\left(r, \sigma^m(s), \sigma^q(t)\right)}{\Delta r^n \Delta s^m} \Delta s \Delta r. \tag{12}$$

Δ −integrating over $t \in [a_3, b_3]$

$$\int_{a_1}^{b_1} \int_{a_2}^{b_2} \int_{a_3}^{b_3} g\left(\sigma^n(r), \sigma^m(s), \sigma^q(t)\right) \Delta t \Delta s \Delta r = \sum_{k=0}^{n-1} \sum_{l=0}^{m-1} X_{k+1}(x) Y_{l+1}(y)$$

$$\times \int_{a_3}^{b_3} \frac{\partial^{k+1} g\left(\sigma^{n-k-1}(x), \sigma^m(y), \sigma^q(t)\right)}{\Delta x^k \Delta y^l} \Delta t + (-1)^m \sum_{k=0}^{n-1} X_{k+1}(x) \int_{a_2}^{b_2} \int_{a_3}^{b_3} Q_m(y,s)$$

$$\times \frac{\partial^{k+m} g\left(\sigma^{n-k-1}(x), \sigma^m(s), \sigma^q(t)\right)}{\Delta x^k \Delta s^m} \Delta t \Delta s + (-1)^n \sum_{l=0}^{m-1} Y_{l+1}(y) \int_{a_1}^{b_1} \int_{a_3}^{b_3} P_n(x,r)$$

$$\times \frac{\partial^{n+1} g\left(r, \sigma^m(y), \sigma^q(t)\right)}{\Delta r^n \Delta y^l} \Delta t \Delta r + (-1)^{m+n} \int_{a_1}^{b_1} \int_{a_2}^{b_2} \int_{a_3}^{b_3} P_n(x,r) Q_m(y,s)$$

$$\times \frac{\partial^{n+m} g\left(r, \sigma^m(s), \sigma^q(t)\right)}{\Delta r^n \Delta s^m} \Delta t \Delta s \Delta r. \tag{13}$$

Where,

$$\int_{a_3}^{b_3} \frac{\partial^{k+1} g\left(\sigma^{n-k-1}(x), \sigma^m(y), \sigma^q(t)\right)}{\Delta x^k \Delta y^l} \Delta t = \sum_{p=0}^{q-1} Z_{p+1}(z) \frac{\partial^{k+l+p} g\left(\sigma^{n-k-1}(x), \sigma^m(y), \sigma^q(z)\right)}{\Delta x^k \Delta y^l \Delta z^p}$$

$$+(-1)^q \int_{a_3}^{b_3} S_q(z,t) \frac{\partial^{k+l+q} g\left(\sigma^{n-k-1}(x), \sigma^m(y), \sigma^q(t)\right)}{\Delta x^k \Delta y^l \Delta t^q} \Delta t. \tag{14}$$

$$\int_{a_3}^{b_3} \frac{\partial^{k+m} g\left(\sigma^{n-k-1}(x), \sigma^m(s), \sigma^q(t)\right)}{\Delta x^k \Delta s^m} \Delta t = \sum_{p=0}^{q-1} Z_{p+1}(z) \frac{\partial^{k+m+p} g\left(\sigma^{n-k-1}(x), \sigma^m(s), \sigma^q(z)\right)}{\Delta x^k \Delta s^m \Delta z^p}$$

$$+(-1)^q \int_{a_3}^{b_3} S_q(z,t) \frac{\partial^{k+m+q} g\left(\sigma^{n-k-1}(x), \sigma^m(s), \sigma^q(t)\right)}{\Delta x^k \Delta s^m \Delta t^q} \Delta t. \tag{15}$$

$$\int_{a_3}^{b_3} \frac{\partial^{n+l} g\left(r, \sigma^m(y), \sigma^q(t)\right)}{\Delta r^n \Delta y^l} \Delta t = \sum_{p=0}^{q-1} Z_{p+1}(z) \frac{\partial^{n+l+p} g\left(r, \sigma^m(y), \sigma^q(z)\right)}{\Delta r^n \Delta y^l \Delta z^p}$$

$$+(-1)^q \int_{a_3}^{b_3} S_q(z,t) \frac{\partial^{n+l+q} g\left(r, \sigma^m(y), \sigma^q(t)\right)}{\Delta r^n \Delta y^l \Delta t^q} \Delta t. \tag{16}$$

And

$$\int_{a_3}^{b_3} \frac{\partial^{n+m} g\left(r, \sigma^m(s), \sigma^q(t)\right)}{\Delta r^n \Delta s^m} \Delta t = \sum_{p=0}^{q-1} Z_{p+1}(z) \frac{\partial^{n+m+p} g\left(r, \sigma^m(s), \sigma^q(z)\right)}{\Delta r^n \Delta s^m \Delta z^p}$$

$$+(-1)^q \int_{a_3}^{b_3} S_q(z,t) \frac{\partial^{n+m+q} g\left(r, \sigma^m(s), \sigma^q(t)\right)}{\Delta r^n \Delta s^m \Delta t^q} \Delta t. \tag{17}$$

From (13) – (17), we have

$$\int_{a_1}^{b_1} \int_{a_2}^{b_2} \int_{a_3}^{b_3} g\left(\sigma^n(r), \sigma^m(s), \sigma^q(t)\right) \Delta t \Delta s \Delta r = \sum_{k=0}^{n-1} \sum_{l=0}^{m-1} \sum_{p=0}^{q-1} X_{k+1}(x) Y_{l+1}(y) Z_{p+1}(z)$$

$$\times \frac{\partial^{k+l+p} g\left(\sigma^{n-k-1}(x), \sigma^m(y), \sigma^q(z)\right)}{\Delta x^k \Delta y^l \Delta z^p} + (-1)^q \sum_{k=0}^{n-1} \sum_{l=0}^{m-1} X_{k+1}(x) Y_{l+1}(y) \int_{a_3}^{b_3} S_q(z,t)$$

$$\times \frac{\partial^{k+l+q} g\left(\sigma^{n-k-1}(x), \sigma^m(y), \sigma^q(t)\right)}{\Delta x^k \Delta y^l \Delta t^q} \Delta t + (-1)^m \sum_{k=0}^{n-1} \sum_{p=0}^{q-1} X_{k+1}(x) Z_{p+1}(z) \int_{a_2}^{b_2} Q_m(y,s)$$

$$\times \frac{\partial^{k+m+p} g\left(\sigma^{n-k-1}(x), \sigma^m(s), \sigma^q(z)\right)}{\Delta x^k \Delta s^m \Delta z^p} \Delta s + (-1)^{m+q} \sum_{k=0}^{n-1} X_{k+1}(x)$$

$$\times \int_{a_2}^{b_2} \int_{a_3}^{b_3} Q_m(y,s) S_q(z,t) \frac{\partial^{k+m+q} g\left(\sigma^{n-k-1}(x), \sigma^m(s), \sigma^q(t)\right)}{\Delta x^k \Delta s^m \Delta t^q} \Delta t \Delta s + (-1)^n$$

$$\times \sum_{l=0}^{m-1} \sum_{p=0}^{q-1} Y_{l+1}(y) Z_{p+1}(z) \int_{a_1}^{b_1} P_n(x,r) \frac{\partial^{n+l+p} g\left(r, \sigma^m(y), \sigma^q(z)\right)}{\Delta r^n \Delta y^l \Delta z^p} \Delta r + (-1)^{n+q}$$

$$\times \sum_{l=0}^{m-1} Y_{l+1}(y) \int_{a_1}^{b_1} \int_{a_3}^{b_3} P_n(x,r) S_q(z,t) \frac{\partial^{n+l+q} g\big(r,\sigma^m(y),\sigma^q(t)\big)}{\Delta r^n \Delta y^l \Delta t^q} \Delta t \Delta r + (-1)^{m+n}$$

$$\times \sum_{p=0}^{q-1} Z_{P+1}(z) \int_{a_1}^{b_1} \int_{a_2}^{b_2} P_n(x,r) Q_m(y,s) \frac{\partial^{n+m+p} g\big(r,\sigma^m(s),\sigma^q(z)\big)}{\Delta r^n \Delta s^m \Delta z^p} \Delta s \Delta r$$

$$+(-1)^{n+m+q} \int_{a_1}^{b_1} \int_{a_2}^{b_2} \int_{a_3}^{b_3} P_n(x,r) Q_m(y,s) S_q(z,t) \frac{\partial^{n+m+q} g\big(r,\sigma^m(s),\sigma^q(t)\big)}{\Delta r^n \Delta s^m \Delta t^q} \Delta t \Delta s \Delta r. \quad (18)$$

The relation (6) follows from (18).

Remark 2. Relation (18) generalizes the relation (2.5) in [14], and for $\mathbb{T} = \boldsymbol{R}$it is re-captured. By setting $x \mapsto \frac{a_1+b_1}{2}$; $y \mapsto \frac{a_2+b_2}{2}$ and $z \mapsto \frac{a_3+b_3}{2}$in (18), we get the generalization of [14, Corollary 2.3] and for $\mathbb{T} = \boldsymbol{R}$[14, Corollary 2.3] is re-captured. For $\mathbb{T} = \boldsymbol{R}$ and by using the different norms such as L_1, L_∞ and L_α , $\alpha > 1$, norms and some mathematical calculations, from (18) [14, Theorem 2.4] is re-captured. Similarly, by setting $\mathbb{T} = \boldsymbol{R}$, we re-capture [14, Corollaries 2.6, 2.7] at the respective boundary points.

Corollary 1. Let $a_i, b_i \in \mathbb{T},\ 1 \le i \le 3$; let $g: \prod_{i=1}^{3} [a_i, b_i]_{\mathbb{T}} \to \boldsymbol{R}$be rd-continuous function such that the partial derivatives $\frac{\partial^{k+l+p} g(.,.,.)}{\Delta r^k \Delta s^t \Delta t^p}, 0 \le k \le 1; 0 \le l \le 1; 0 \le p \le 1$ exist and are continuous on $\prod_{i=1}^{3} [a_i, b_i]_{\mathbb{T}}$. Then, for $(x,y,z) \in \prod_{i=1}^{3} [a_i, b_i]$, the following inequality holds:

$$\Bigg| \int_{a_1}^{b_1} \int_{a_2}^{b_2} \int_{a_3}^{a_3} g\big(\sigma(r),\sigma(s),\sigma(t)\big) \Delta t \Delta s \Delta r - X_1(x) Y_1(y) Z_1(z) g\big(x,\sigma(y),\sigma(z)\big)$$

$$+ X_1(x) Y_1(y) \int_{a_3}^{b_3} S_1(z,t) \frac{\partial g\big(x,\sigma(y),\sigma(t)\big)}{\Delta t} \Delta t$$

$$+ X_1(x) Z_1(z) \int_{a_2}^{b_2} Q_1(y,s) \frac{\partial g\big(x,\sigma(s),\sigma(z)\big)}{\Delta s} \Delta s$$

$$- X_1(x) \int_{a_2}^{b_2} \int_{a_3}^{b_3} Q_1(y,s) S_1(z,t) \frac{\partial^2 g\big(x,\sigma(s),\sigma(t)\big)}{\Delta s \Delta t} \Delta s \Delta t$$

$$+ Y_1(y) Z_1(z) \int_{a_1}^{b_1} P_1(x,r) \frac{\partial g\big(r,\sigma(y),\sigma(z)\big)}{\Delta r} \Delta r$$

$$- Y_1(y) \int_{a_1}^{b_1} \int_{a_3}^{b_3} P_1(x,r) S_1(z,t) \frac{\partial^2 g\big(r,\sigma(y),\sigma(t)\big)}{\Delta r \Delta t} \Delta t \Delta r -$$

$$- Z_1(z) \int_{a_1}^{b_1} \int_{a_2}^{b_2} P_1(x,r) Q_1(y,s) \frac{\partial^2 g\big(r,\sigma(s),\sigma(z)\big)}{\Delta r \Delta s} \Delta s \Delta r \Bigg|$$

$$\le \left\| \frac{\partial^3 g}{\Delta r \Delta s \Delta t} \right\|_\infty \int_{a_1}^{b_1} \int_{a_2}^{b_2} \int_{a_3}^{a_3} |P_1(x,r) Q_1(y,s) S_1(z,t)| \Delta t \Delta s \Delta r. \quad (19)$$

Remark 3. Relation (19) is the generalized Ostrowski type inequality in triple integral on general time scales.

The following result is the classical Ostrowski type inequality for triple integrals.

Corollary 2.(continuous case) Let $\mathbb{T} = \boldsymbol{R},\ n = m = q = 1;\ k = l = p = 0$ and

$$\left\|g'''_{x,y,z}\right\|_\infty = \sup_{(x,y,z)\in(a_1,b_1)\times(a_2,b_2)\times(a_3,b_3)} \left|\frac{\partial^3 g(x,y,z)}{\partial x\partial y\partial z}\right| < \infty.$$

Then (19) takes the form:

$$\Bigg|\int_{a_1}^{b_1}\int_{a_2}^{b_2}\int_{a_3}^{b_3} g(r,s,t)dtdsdr - (b_3-a_3)(b_2-a_2)(b_1-a_1)g(x,y,z)$$
$$+(b_3-a_3)(b_1-a_1)\int_{a_2}^{b_2} g(x,s,z)ds + (b_2-a_2)(b_1-a_1)\int_{a_3}^{b_3} g(x,y,t)dt$$
$$+(b_3-a_3)(b_2-a_2)\int_{a_1}^{b_1} g(r,y,z)dr - (b_3-a_3)\int_{a_1}^{b_1}\int_{a_2}^{b_2} g(r,s,z)dsdr$$
$$-(b_2-a_2)\int_{a_1}^{b_1}\int_{a_3}^{b_3} g(r,y,t)dtdr - (b_1-a_1)\int_{a_2}^{b_2}\int_{a_3}^{b_3} g(x,s,t)dtds\Bigg|$$
$$\le \left\|g'''_{r,s,t}\right\|_\infty \frac{(r-a_1)^2+(b_1-r)^2}{2}\frac{(s-a_2)^2+(b_2-s)^2}{2}\frac{(t-a_3)^2+(b_3-t)^2}{2}. \qquad (20)$$

4. APPLICATIONS FOR GENERALIZED POLYNOMIAL

Example 1. (Discrete case) Let $\mathbb{T} = \mathbb{Z}$. Then, we have

$$h_k(t,s) = \frac{(t-s)^{(k)}}{k!} = (-1)^k\frac{(s-t+k)^{(k)}}{k!}$$

for $s,t \in \mathbb{T}$ and $k \in \mathbb{N}$, where the usual factorial function, (k), is defined as $n^{(k)} = \frac{n!}{k!}$ for $k \in \mathbb{N}$ and $(n)^{(0)} = 1$ for $n \in \mathbb{Z}$. In this case the inequality (6) reduces to the following inequality:

$$\Bigg|\sum_{r=a_1}^{b_1-1}\sum_{s=a_2}^{b_2-1}\sum_{t=a_3}^{b_3-1} g(r+n,s+m,q+t) - \sum_{k=0}^{n-1}\sum_{l=0}^{m-1}\sum_{p=0}^{q-1}\sum_{\alpha=0}^{p}\sum_{\beta=0}^{l}\sum_{\Gamma=0}^{k}(-1)^{k+l+p-\alpha-\beta-\Gamma}$$
$$\times\binom{p}{\alpha}\binom{l}{\beta}\binom{k}{\Gamma}\frac{(b_1-x+k+1)^{(k+1)}+(-1)^k(x-a_1)^{(k+1)}}{(k+1)!}$$
$$\times\frac{(b_2-y+l+1)^{(l+1)}+(-1)^l(y-a_2)^{(l+1)}}{(l+1)!}\frac{(b_3-z+p+1)^{(p+1)}+(-1)^p(z-a_3)^{(p+1)}}{(p+1)!}$$
$$\times g(x+n-k-1+\Gamma, \mathrm{y}+\mathrm{m}+\beta, \mathrm{z}+\mathrm{q}+\alpha) - \sum_{k=0}^{n-1}\sum_{l=0}^{m-1}\sum_{\delta=0}^{q}\sum_{\beta=0}^{l}\sum_{\Gamma=0}^{k}\sum_{t=a_3}^{b_3-1}(-1)^{k+l-\delta-\beta-\Gamma}$$
$$\times\binom{q}{\delta}\binom{l}{\beta}\binom{k}{\Gamma}\frac{(b_1-x+k+1)^{(k+1)}+(-1)^k(x-a_1)^{(k+1)}}{(k+1)!}S_q(z,t)$$
$$\times\frac{(b_2-y+l+1)^{(l+1)}+(-1)^l(y-a_2)^{(l+1)}}{(l+1)!}g(x+n-k-1+\Gamma, \mathrm{y}+\mathrm{m}+\beta, \mathrm{t}+\mathrm{q}+\delta)$$
$$-\sum_{k=0}^{n-1}\sum_{p=0}^{q-1}\sum_{s=a_2}^{b_2-1}\sum_{\alpha=0}^{p}\sum_{\eta=0}^{m}\sum_{\Gamma=0}^{k}(-1)^{k+p-\alpha-\eta-\Gamma}\binom{p}{\alpha}\binom{m}{\eta}\binom{k}{\Gamma}\frac{(b_1-x+k+1)^{(k+1)}+(-1)^k(x-a_1)^{(k+1)}}{(k+1)!}$$

$$\times \frac{(b_3-z+p+1)^{(p+1)}+(-1)^p(z-a_3)^{(p+1)}}{(p+1)!} g(x+n-k-1+\Gamma, s+m+\eta, z+q+\delta)$$

$$-\sum_{k=0}^{n-1}\sum_{s=a_2}^{b_2-1}\sum_{t=a_3}^{b_3-1}\sum_{\delta=0}^{q}\sum_{\eta=0}^{m}\sum_{\Gamma=0}^{k}(-1)^{k-\delta-\eta-\Gamma}\frac{(b_1-x+k+1)^{(k+1)}+(-1)^k(x-a_1)^{(k+1)}}{(k+1)!}$$

$$\times g(x+n-k-1+\Gamma, s+m+\eta, t+q+\delta)Q_m(y,s)S_q(z,t)$$

$$-\sum_{l=0}^{m-1}\sum_{p=0}^{q-1}\sum_{r=a_1}^{b_1-1}\sum_{\psi=0}^{n}\sum_{\beta=0}^{l}\sum_{\alpha=0}^{p}(-1)^{l+p-\psi-\beta-\alpha}P_n(x,r)\frac{(b_2-y+l+1)^{(l+1)}+(-1)^l(y-a_2)^{(l+1)}}{(l+1)!}$$

$$\times \frac{(b_3-z+p+1)^{(p+1)}+(-1)^p(z-a_3)^{(p+1)}}{(p+1)!} g(r+\psi, y+m+\beta, z+q+\alpha)$$

$$-\sum_{l=0}^{m-1}\sum_{r=a_1}^{b_1-1}\sum_{t=a_3}^{b_3-1}\sum_{\psi=0}^{n}\sum_{\beta=0}^{l}\sum_{\delta=0}^{q}(-1)^{l-\psi-\beta-\delta}P_n(x,r)S_q(z,t)\frac{(b_2-y+l+1)^{(l+1)}+(-1)^l(y-a_2)^{(l+1)}}{(l+1)!}$$

$$\times g(r+\psi, y+m+\beta, t+q+\delta)-\sum_{p=0}^{q-1}\sum_{r=a_1}^{b_1-1}\sum_{s=a_2}^{b_2-1}\sum_{\alpha=0}^{p}\sum_{\eta=0}^{m}\sum_{\psi=0}^{n}(-1)^{p-\psi-\eta-\alpha}P_n(x,r)Q_m(y,s)$$

$$\times \frac{(b_3-z+p+1)^{(p+1)}+(-1)^p(z-a_3)^{(p+1)}}{(p+1)!} g(r+\psi, s+m+\eta, z+q+\alpha)\Bigg|$$

$$\leq M\sum_{r=a_1}^{b_1-1}\sum_{s=a_2}^{b_2-1}\sum_{t=a_3}^{b_3-1}\left|P_n(x,r)Q_m(y,s)S_q(z,t)\right|.$$

Where, M =maximum value of the absolute value of

$$\sum_{\psi=0}^{n}\sum_{\eta=0}^{l}\sum_{\delta=0}^{q}(-1)^{n+m+q-\psi-\eta-\delta}g(r+\eta, s+m+\eta, t+q+\delta),$$

over$[a_1, b_1-1]_{\mathbb{Z}} \times [a_2, b_2-1]_{\mathbb{Z}} \times [a_3, b_3-1]_{\mathbb{Z}}$

Example 2. (Quantum calculus case) Let $\mathbb{T}=q^{\mathbb{N}_0}$ with $q>1$.Then in this case Ostrowski inequality (19) takes the following form:

$$\Bigg|\sum_{r=0}^{log_q^{(b_1/(q\alpha_1))}}\sum_{s=0}^{log_q^{(b_2/(q\alpha_2))}}\sum_{t=0}^{log_q^{(b_3/(q\alpha_3))}} g(a_1q^{r+1}, a_2q^{s+1}, a_3q^{t+1})-(b_3-a_3)(b_2-a_2)$$

$$\times (b_1-a_1)g(x,qy,qz)+(b_2-a_2)(b_1-a_1)\sum_{t=0}^{log_q^{(b_3/(q\alpha_3))}} S_1(z,a_3q^t)$$

$$\times \frac{g(x,qy,q^2t)-g(x,qy,qt)}{q(q-1)t}+(b_3-a_3)(b_1-a_1)\sum_{s=0}^{log_q^{(b_2/(q\alpha_2))}} Q_1(y,a_2q^s)$$

$$\times \frac{g(x,q^2s,qz)-g(x,qs,qz)}{q(q-1)s} + (b_3-a_3)(b_2-a_2) \sum_{r=0}^{log_q^{(b_1/(q\alpha_1))}} P_1(x,a_1q^r)$$

$$\times \frac{g(qr,qy,qz)-g(r,qy,qz)}{(q-1)r} - (b_1-a_1) \sum_{s=0}^{log_q^{(b_2/(q\alpha_2))}} \sum_{t=0}^{log_q^{(b_3/(q\alpha_3))}} Q_1(y,a_2q^s)\, S_1(z,a_3q^t)$$

$$\times \frac{g(x,sq^2,tq^2)-g(x,sq^2,qt)-g(x,sq,tq^2)+g(x,sq,tq)}{st(q^2-q)^2} - (b_2-a_2)$$

$$\times \sum_{r=0}^{log_q^{(b_1/(q\alpha_1))}} \sum_{t=0}^{log_q^{(b_3/(q\alpha_3))}} \frac{g(qr,qy,tq^2)-g(r,qy,tq^2)-g(qr,qy,qt)+g(r,qy,tq)}{rqt(q-1)^2}$$

$$\times P_1(x,a_1q^r)\ S_1(z,a_3q^t) - (b_3-a_3) \sum_{r=0}^{log_q^{(b_1/(q\alpha_1))}} \sum_{s=0}^{log_q^{(b_2/(q\alpha_2))}} P_1(x,a_1q^r)\, Q_1(y,a_2q^s)$$

$$\times \frac{g(qr,sq^2,qz)-g(qr,qs,qz)-g(r,sq^2,qz)+g(r,qs,qz)}{rqs(q-1)^2}\Bigg|$$

$$\leq M \sum_{r=0}^{log_q^{(b_1/(q\alpha_1))}} \sum_{s=0}^{log_q^{(b_2/(q\alpha_2))}} \sum_{t=0}^{log_q^{(b_3/(q\alpha_3))}} |P_1(x,a_1q^r)Q_1(y,a_2q^s)S_1(z,a_3q^t)|.$$

Where, M is the maximum value of the absolute value of

$$\frac{g(qr,qs,qt)+g(r,s,qt)+g(r,qs,t)+g(qr,s,t)}{rst(q-1)^3} - \frac{g(r,qs,qt)+g(qr,s,qt)+g(qr,qs,t)+g(r,s,t)}{rst(q-1)^3},$$

over $[a_1,b_1/q]_{q^{\mathbb{N}_0}} \times [a_2,b_2/q]_{q^{\mathbb{N}_0}} \times [a_3,b_3/q]_{q^{\mathbb{N}_0}}$.

5. REFERENCES

1. Anastassiou, G.A. Ostrowski type inequalities. *Proceedings of the American Mathematical Society* 123 (12): 3775-3781(1995).
2. Barnett, N.S. & S.S. Dragomir. An Ostrowski type inequality for double integrals andapplication for cubature formulae. *Soochow Jurnal of Mathematics* (in press).
3. Ceron, P., S.S. Dragomir & J. Roumeliotis. Some Ostrowski type inequalities for n-times differentiable mappings and applications. *Demonstration Mathematica* 32(4): 697-712 (1999).
4. Dragomir, S.S., N.S. Barnett & P. Cerone. An Ostrowski type inequality for double integrals in terms of Lp-Norms and applications in Numerical integrations. *Anal. Num. Theor. Approx (Romani*a) (in press).
5. Fink, A.M. Bounds on the derivation of a function from its averages, Czechoslovak Math. J. 42: 289-310 (1992).
6. Hanna, G., S.S. Dragomir & P. Cerone. A general Ostrowski type inequality for doubleintegrals. *Tamsui Oxford Journal of Mathematical Sciences* 18(1): 1-16 (2002).
7. Hussain, S., M.A. Latif & M. Alomari. Generalized Double Integral Ostrowski type inequality on Time Scales. *Applied Mathematics Letters* 24: 1461-1467 (2011).
8. Hilger, S. Ein Maβkettenkalk$\ddot{u}$l mit anwendung auf zentrumsmannigfaltigkeiten. PhD thesis, Universitaat W$\ddot{u}$rzburg, W$\ddot{u}$urzburg, Germany (1988).

9. Ostrowski, A. ***Ü***ber die Absolutabweichungeinerdifferentiebarenfunktin von ihrenintergralmittelwert. *Commentarii Mathematici Helvetici* 10: 226-227 (1938).
10. Pachpatte, B.G. Discrete inequalities in three independent variables. *Demonstratio Mathematica* 31: 849-854 (1999).
11. Pachpatte, B.G. On an inequality of Ostrowski type in three independent variables. *Journal of Mathematical Analysis and Applications* 249: 583-591 (2000).
12. Srivastava, H.M., K. -L. Tseng, S. -J. Tseng & J. -C. Lo. Some weighted Opial-type inequalities on time scale. *Taiwanese Journal of Mathematics* 14: 107-122 (2010).
13. Srivastava, H.M., K. -L. Tseng, S. -J. Tseng and J. -C. Lo, Some generalizations of Maroni's inequality on time scales. *Mathematical Inequalities and Applications* 14: 469-480 (2011).
14. Sofo, A. An Integral approximation in three variables. *Journal of Inequalities in Pure and Applied Mathematics* 3(4) (2003).
15. Bohner, M. & A. Peterson. *Advances in Dynamic Equations on Time Scales*. Birkhäuser, Boston, MA (2003).
16. Bohner, M. & T. Matthews. Ostrowski inequalities on time scales. *Jouranl of Inequalities in Pure and Applied Mathematics* 9: Article 6, 8 pp (2008).
17. Dinu, C. Ostrowski type inequalities on time scales. *Annals of the University of Craiova – Ser. Mat. Inform.* 34: 43-58 (2007).
18. Liu, W.J. & Q.-A.Ngô. An Ostrowski-Grüss type inequality on time scales. *Computers and Mathematics with Applications* 58: 1207-1210 (2209).
19. Liu, W.J. & Q.-A.Ngô. A generalization of Ostrowski inequality on time scales for k points. *Applied Mathematics and Computation* 203: 754-760 (2008).
20. Tuna, A. & D. Daghan. Generalization of Ostrowski and Ostrowski-Grüss type inequalities on time scales. *Computers and Mathematics with Applications* 60: 803-811 (2010).
21. Tuna, A., Y. Jiang, & W.J. Liu. Weighted Ostrowski, Ostrowski-Grüss and Ostrowski- Čebyhev type inequalities on time scales. *Publicationes Mathematicae-Debercen* 81: 81-102 (2012).

Proceedings of the Pakistan Academy of Sciences 51 (3): 259–260 (2014)

ISSN: 0377 - 2969 (print), 2306 - 1448 (online)

Pakistan Academy of Sciences

Obituary

Professor Dr. Kamaluddin Ahmed

Pride of Performance

(1939-2014)

9th June, 2014 was a sad day for us all; colleagues, friends, students and admirers of Dr. Kamal, who breathed his last in the US where he had gone with his family for a short visit. A few days after his arrival in the US, he fell ill with jaundice from which he did not recover. Within three weeks after his diagnosis he was no more with us. It was very saddening and heartbreaking!

May his soul rest in peace.

Dr. Kamal was born in Delhi on 15th April, 1939. He did his masters in Mathematics from Punjab University, his DIC and PhD from Imperial College London (1965) in the field of theoretical particle physics. The 1960s were exciting days in High Energy Physics (HEP) and Prof. Salam's group at Imperial was in the forefront of this activity. A large number of Pakistani students including Kamal were actively engaged in this field at Imperial at the time. These students later became pioneers of HEP in Pakistan at QAU. Dr. Kamal was an outstanding figure in this team.

He established himself as an excellent teacher and a competent research guide. He supervised a large number of M.Phil. and PhD students who admired and loved him. In recognition of his services he was given many prestigious awards and honors.

IMPERIAL COLLEGE LONDON
Theoretical Physics Group, 1963

Dr. Kamal had the unique distinction amongst Pakistani physicists, who did his postdoctoral at the world famous JINR, Dubna, Moscow (1966-67) and later at DESY, Hamburg (1974-75). His research interests covered many areas of HEP, Astroparticle Physics, Cosmology and the Early Universe. After his retirement from QAU (1999), he served at Hamdard University, COMSATS and very recently at the NCP.

With his demise, we have lost a friend, a colleague, an excellent physicist and above all a fine human being. Personally, it is a loss of a close family friend who shared his many ups and downs in life with us. Our association is spread over a long period of more than 50 years; first at PU where he was my junior, then a fellow research student at Imperial College London and later a colleague at QAU, Islamabad.

I had the opportunity to visit his parents for the first time in 1960-61 when they used to live in Lahore near the railway station Nolakha Chowk. His parents were highly cultured folks who had migrated from Delhi where they were 'jagirdars' and thus called Nawabs, though Dr. Kamal never used this family name. His father was a barrister at law from London but he adopted to serve in education rather than in the legal profession.

Dr. Kamal leaves behind a widow and two sons.

Dr. G. Murtaza, *FPAS*
Distinguished National Professor
Cell: +92-331-4586520

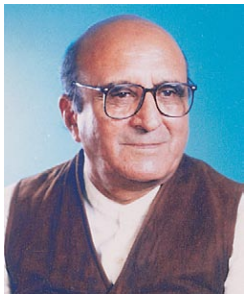
Proceedings of the Pakistan Academy of Sciences 51 (3): 261 (2014)

ISSN: 0377 - 2969 (print), 2306 - 1448 (online)

Pakistan Academy of Sciences

Obituary

Prof. Dr. R. A. Khan Tahirkheli, *S.I.*
(1928 - 2014)



Dr. Rashid Ahmad Khan Tahirkheli, an internationally renowned geologist, was laid to rest in his ancestral graveyard in Ghazi near Turbela, Khyber Pakthunkhawa, on August 14, 2014. Dr. Tahirkheli was born in his native town Ghazi, Hazara on April 4, 1928. After early education at Haripur, he graduated from Aligarh Muslim University, India, and joined the Geological Survey of Pakistan in 1951 as Assistant Geologist. Later on, he went to the University of St Andrews, Scotland, for PhD studies. Upon return in 1964, he joined the University of Peshawar (UoP) as Reader and Chairman of the Department of Geology. In 1969, he became a full Professor.

Dr. Tahirkheli's contributions to the growth and development of Earth Sciences in Pakistan are manifold. He succeeded in establishing the National Center of Excellence in Geology (NCEG) in 1975 of which he remained the founding Director till his retirement in 1988. He became the Vice Chancellor of the UoP in 1982 and successfully completed a five years tenure whereby he brought reforms in the academics of the University and also established new departments. Over the years, these turned out to be amongst the most productive departments of the University. In 1999 he became the Vice Chancellor of Gandhara University, Peshawar, and remained on this post till his last breath. He also remained Professor Emeritus in the NCEG.

Dr. Tahirkheli played a vital role in getting financial support for his young colleagues to pursue higher studies abroad and for equipping the NCEG labs. His enthusiasm, encouragement, support and personal involvement led to a considerable understanding of the complex geology of northern Pakistan – the abode of the mighty Himalaya, Karakoram and Hindukush ranges and one of the most rugged and inaccessible terrains. Dr. Tahirkheli's hard work and remarkable grasp of geology of Pakistan attracted the attention of numerous international geologists for collaborative research. One of the best known Himalayas geologists, Dr. K. S. Valdiya, described Prof. Tahirkheli as "an active and vigorous person, always in the forefront of daring ventures regardless of time, place and age". No wonder he was (1) appointed an Adjunct Professor at Dartmouth College, USA; (2) Member of the Geological & Mineral Coordination Board, Government of Pakistan (GoP); (3) elected as Fellow of the Pakistan Academy of Sciences, (4) an honorary Fellow of the Geological Society of America; (5) awarded Medal for Academic Excellence by the University of Etinboro, USA; and (6) conferred upon *Sitara-e-Imtiaz* by the Government of Pakistan. His publications are of international repute and landmarks for the national and international geologists working in the northern Pakistan.

Dr. Tahirkheli was a cheerful, generous, gracious, and tolerant person, a true friend and a good host. It is a matter of pride for all those to have been associated with him. His death is a big loss to Pakistan and geology. We have been bereaved of our teacher, mentor, and most dependable and caring colleague. May Almighty Allah bless his soul in eternal peace. Amin.

Prof. Dr. M. Tahir Shah
Director,
National Center of Excellence in Geology,
University of Peshawar, Peshawar, Pakistan

Proceedings of the Pakistan Academy of Sciences

Instructions for Authors

Aims and Scope: *Proceedings of the Pakistan Academy of Sciences* is official journal of the Academy, published quarterly, in English. This open access journal publishes research papers in *Engineering Sciences & Technology, Life Sciences, Medical Sciences, and Physical Sciences*. State-of-the-art reviews (~20 pages, supported by recent references) summarizing R&D in a particular area of science, especially in the context of Pakistan, and suggesting further R&D are also considered. Manuscripts undergo double-blind review. Authors are not required to be Fellows or Members of the *Pakistan Academy of Sciences* or citizens of Pakistan.

Manuscript Format
Manuscript may contain **Abstract, Keywords, INTRODUCTION, MATERIALS AND METHODS, RESULTS, DISCUSSION (or RESULTS AND DISCUSSION), CONCLUSIONS, ACKNOWLEDGEMENTS and REFERENCES** *and any other information that the author(s) may consider necessary.* The Manuscript sections must be numbered, i.e., **1. INTRODUCTION, 2. MATERIALS AND METHODS,** and so on.

Manuscripts, in *Times New Roman*, 1.5-spaced (but single-space the Tables), with line numbering and one-inch margins on all sides on A-4 size paper, should not exceed 20 pages including Tables and Figures. Number manuscript pages throughout. The text (in **Font Size 11,** except for the sections mentioned in **Font Size 10)** must be typed in a single column across the paper width. All Tables and Figures must be placed after the text, i.e., after REFERENCES section.

Title Page: For Online submission of the manuscript, Title Page should not be included. However, for submitting as an email attachment, the title page must be as under:
(a) **Title** of the article **(**Capitalize initial letter of each main word; font size 16; **bold**), max 160 characters (no abbreviations or acronyms), depicting article's contents; (b) Author' first name, middle initial and last name (font size 12, **bold**), and professional affiliation (i.e., each author's Department, Institution, Mailing address and Email; but no position titles) (font size 12); (c) Indicate the corresponding author with *; (d) **Short running title**, max 50 characters (font size 10).

The **next Page** should start with **Title** of the Article, followed by entire manuscript.

Headings and Subheadings (font size 11): All flush left

LEVEL-1: **ALL CAPITAL LETTERS; bold**
Level-2: **Capitalize each main word; bold**
Level-3: Capitalize each main word; Bold, Italic
Level-4: *Run-in head*; *Italics*, *in the normal paragraph position. Capitalize the initial word only and end in a colon (i.e., :)*

Abstract (font size 10; max 250 words)**:** Must be self-explanatory**,** stating rationale, objective(s), methodology, main results and conclusions of the study. Abbreviations, if used, must be defined on first mention in the Abstract as well as in the main text. Abstract of review articles may have variable format.
Keywords (font size 10)**:** Three to eight keywords**,** depicting the article.

For Online Submission:

- **Separate submission of Abstract along with Title is required**
- **Author(s)' names and affiliations should not be included in the manuscript or in the Abstract**

INTRODUCTION: Provide a clear and concise statement of the problem, citing relevant recent literature, and objectives of the investigation.

MATERIALS AND METHODS: Provide an adequate account of the procedures or experimental details, including statistical tests (if any), in a concise manner but sufficient enough to replicate the study.

RESULTS: Be clear and concise with the help of appropriate Tables, Figures and other illustrations. Data should not be repeated in Tables and Figures, but must be supported with statistics.

DISCUSSION: Provide interpretation of the RESULTS in the light of previous relevant studies, citing published references.

ACKNOWLEDGEMENTS (font size 10)**:** In a brief statement, acknowledge financial support and other assistance.

REFERENCES (font size 10)**:** Cite references in the text **by number only** in **square brackets**, e.g. "Brown et al [2] reported ..." or "... as previously described [3, 6–8]", and list them in REFERENCES section, in the order of citation in the text, Tables and Figures (not alphabetically). Only published (and accepted for publication) journal articles, books, and book chapters qualify for REFERENCES.

List of REFERENCES must be prepared as under:

a. **Journal Articles** *(Name of journals must be stated in full)*

1. Golding, I. Real time kinetics of gene activity in individual bacteria. *Cell* 123**:** 1025–1036 (2005).
2. Bialek, W. & S. Setayeshgar. Cooperative sensitivity and noise in biochemical signaling. *Physical Review Letters* 100**:** 258–263 (2008).
3. Kay, R.R. & C.R.L. Thompson. Forming patterns in development without morphogen gradients: differentiation and sorting. *Cold Spring Harbor Perspectives in Biology* 1: doi: 10.1101/cshperspect.a001503 (2009).

b. **Books**

4. Luellen, W.R. *Fine-Tuning Your Writing*. Wise Owl Publishing Company, Madison, WI, USA (2001).
5. Alon, U. & D.N. Wegner (Ed.). *An Introduction to Systems Biology: Design Principles of Biological Circuits*. Chapman & Hall/CRC, Boca Raton, FL, USA (2006).

c. **Book Chapters**

6. Sarnthein, M.S. & J.D. Stanford. Basal sauropodomorpha: historical and recent phylogenetic developments. In: *The Northern North Atlantic: A Changing Environment.* Schafer, P.R. & W. Schluter (Ed.), Springer, Berlin, Germany, p. 365–410 (2000).
7. Smolen, J.E. & L.A. Boxer. Functions of Europhiles. In: *Hematology, 4th ed*. Williams, W.J., E. Butler & M.A. Litchman (Ed.), McGraw Hill, New York, USA, p. 103–101 (1991).

Tables, with concise but self-explanatory headings must be numbered according to the order of citation (like **Table 1**, **Table 2**). Round off data to the nearest three significant digits. Provide essential explanatory footnotes, with superscript letters or symbols keyed to the data. Do not use vertical or horizontal lines, except for separating column heads from the data and at end of the Table.

Figures may be printed in two sizes: column width of 8.0 cm or page width of 16.5 cm; number them as **Fig. 1**, **Fig. 2**, … in the order of citation in the text. Captions to Figures must be concise but self-explanatory. Laser printed line drawings are acceptable. Do not use lettering smaller than 9 points or unnecessarily large. Photographs must be of high quality. A scale bar should be provided on all photomicrographs.

Declaration: Provide a declaration that: (i) the results are original; (ii) the same material is neither published nor under consideration elsewhere; (iii) approval of all authors has been obtained; and, (iv) in case the article is accepted for publication, its copyright will be assigned to *Pakistan Academy of Sciences*. Authors must obtain permission to reproduce copyrighted material from other sources.

Reviewers: Authors may suggest four relevant reviewers, two local and two from scientifically advanced countries.

Online Manuscript Submission:
Proceedings of the Pakistan Academy of Sciences is using a web-enabled online manuscript submission and review system, which offers authors the option to track the progress of manuscripts in real time.
For online submission, authors are required to register at website of the Academy (www.paspk.org) by clicking the "Journal and Publications" tab.

Manuscripts must be submitted in Microsoft Word (.doc or .docx format; **pdf** files not acceptable). Figures can be submitted in Word format, TIFF, GIF, JPEG, EPS, PPT.

NOTE: In case of online submission, authors are not required to submit the manuscript as email attachment.
In case of any difficulty while submitting your manuscript, please get in touch with:

Editor-in-Chief
Pakistan Academy of Sciences
3-Constitution Avenue, G-5/2, Islamabad, Pakistan
Email: pas.editor@gmail.com
Tel: +92-51-920 7140
Website: www.paspk.org

Proceedings
OF THE PAKISTAN ACADEMY OF SCIENCES

CONTENTS

Volume 51, No. 3, September 2014 Page

PAKISTAN ACADEMY OF SCIENCES, ISLAMABAD, PAKISTAN
HEC Recognized, Category X; PM&DC Recognized